MATHEMATICS SYLLABUS

Pre-University

H2 Mathematics

Implementation starting with
2016 Pre-University One Cohort

Ministry of Education
SINGAPORE

CONTENTS

1. INTRODUCTION

Importance of Mathematics

Mathematics contributes to the developments and understanding in many disciplines. It is used extensively to model the real world, create new products and services and support data-driven decisions. A good foundation in mathematics and a keen appreciation of its potential give one a competitive edge over others.

Discipline of Mathematics

Mathematics is a study about quantities, space, patterns, relationships, chance and abstractions. Mathematical knowledge is established through rigorous proofs, derived from axioms and definitions through logical argument and reasoning. Mathematical statements or claims should be challenged and remain as conjectures until they are proven to be true.

Mathematics can be seen as a language. It is used to express, communicate and share ideas, within the scientific communities as well as with the general public. It has its own set of notations, symbols, and terminologies. It is a language that strives to be precise and concise.

The applications of mathematics transcend its own boundary, into the daily life, the real world and other disciplines. It is more than just computations. Mathematics is a powerful tool to model real world phenomena. But it has its limitations, as often mathematical models cannot capture all the complexities of real world.

Learning of Mathematics

The learning of mathematics should honour the nature of the discipline and its practices. Students should therefore learn to justify their solutions, give reasons to support their conclusions and prove mathematical statements. They should also learn to communicate mathematically, construct and discuss mathematical statements, and use the language of mathematics to develop and follow a logical chain of reasoning. In applying mathematics to solve real world problems, they should learn to formulate models, be aware of the limitations of these models and exercise care in the interpretation of mathematics solutions. Such learning experiences will provide students a glimpse of what being a mathematician is like and what mathematics is about.

Mathematics at the A-Level

In Singapore, mathematics education at the A-level plays an important role in laying the foundation for building a pool of highly skilled and analytical workforce, especially in STEM-related areas. From the period of rapid industrialisation in the 80’s to the current day of knowledge intensive industries, it continues to be highly valued by stakeholders and students preparing for tertiary education. Although mathematics is an optional subject at the A-level, it is offered by nearly all students.

The purpose of learning mathematics at the A-level is two-fold. Firstly, it provides students, regardless of the intended course of study at the university, with a useful set of tools and problem solving skills to support their tertiary study. Secondly, learning mathematics exposes students to a way of thinking that complements the ways of thinking developed through other disciplines. This contributes to the development of a well-rounded individual who is able to think deeply, broadly and differently about problems and issues.

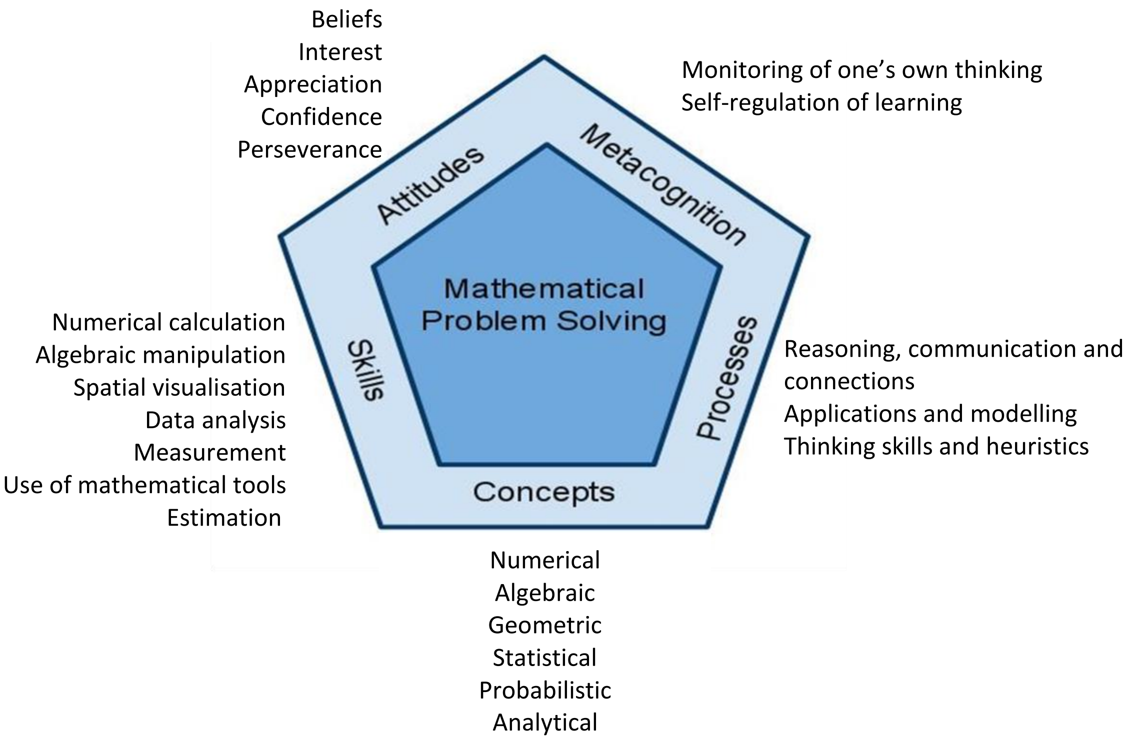
A suite of syllabuses is available to students at the A-level. The syllabuses are:

- H1 Mathematics;
- H2 Mathematics;
- H2 Further Mathematics; and
- H3 Mathematics.

The suite of syllabuses is designed for different profiles of students, to provide them with options to learn mathematics at different levels, and to varying breadth, depth or specialisation so as to support their progression to their desired choice of university courses.

Mathematics Framework

The Mathematics Framework sets the direction for curriculum and provides guidance in the teaching, learning, and assessment of mathematics. The central focus is mathematical problem solving, that is, using mathematics to solve problems. The curriculum stresses *conceptual understanding*, *skills proficiency* and *mathematical processes*, and gives due emphasis to *attitudes* and *metacognition*. These five components are inter-related.



- *Concepts*

At the A-level, students continue to study concepts and skills in the major strands of mathematics, which provide the building blocks for the learning of advanced mathematics, with varying breadth and depth depending on the syllabuses. These major strands include Algebra, Calculus, Vectors, and Probability and Statistics, which are rich in applications within mathematics and in other disciplines and the real world. These content categories are connected and interdependent.

- *Skills*

Mathematical skills refer to *numerical calculation*, *algebraic manipulation*, *spatial visualisation*, *data analysis*, *measurement*, *use of mathematical tools*, and *estimation*. The skills are specific to mathematics and are important in the learning and application of mathematics. In today's classroom, these skills also include the abilities to use spreadsheets and other software to learn and do mathematics.

- *Processes*

Mathematical processes refer to the skills involved in acquiring and applying mathematical knowledge. These include *reasoning, communication and connections*, *applications and modelling*, and *thinking skills and heuristics* that are important in mathematics.

Reasoning, communication and connections

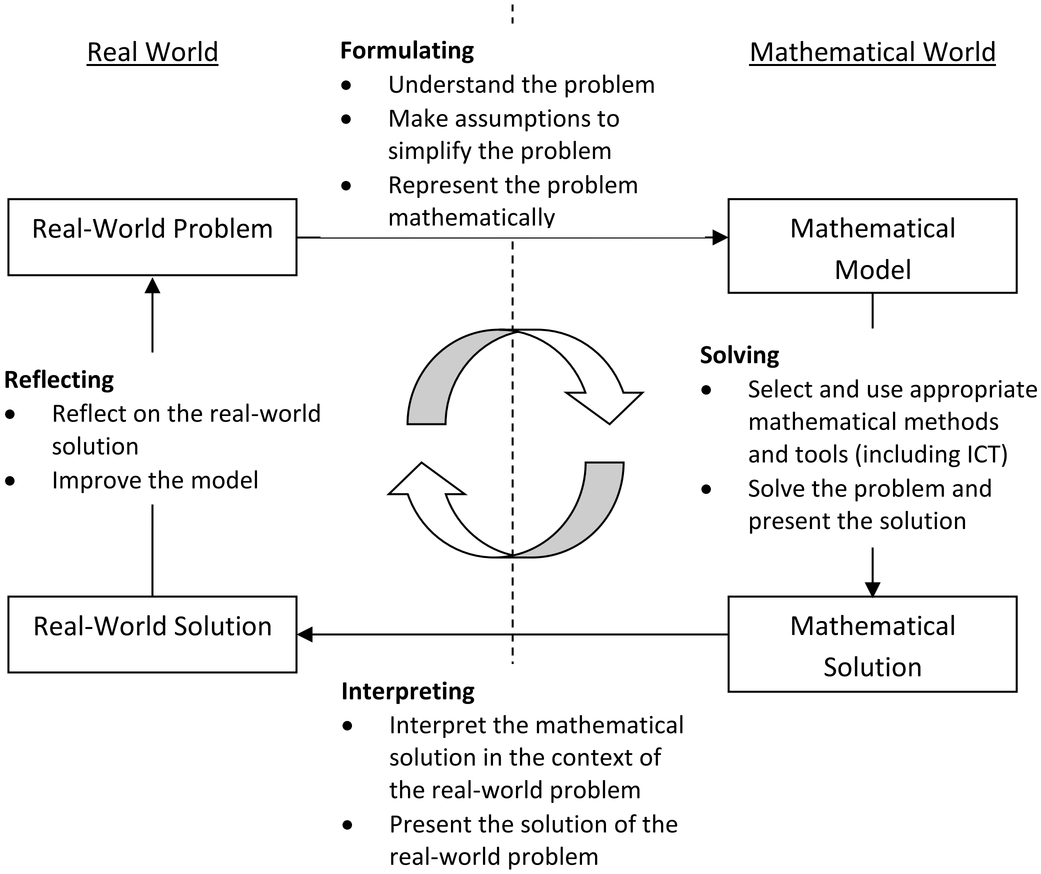
- Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments.
- Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically.
- Connections refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and the real world.

Applications and modelling

Exposing students to applications and modelling enhances their understanding and appreciation of mathematics. Mathematical modelling is the process of formulating and improving a mathematical model[1] to represent and solve real-world problems. Through mathematical modelling, students learn to deal with complexity and ambiguity by simplifying and making reasonable assumptions, select and apply appropriate mathematical concepts and skills that are relevant to the problems, and interpret and evaluate the solutions in the context of the real-world problem.

[1] A mathematical model is a mathematical representation or idealisation of a real-world situation. It can be as complicated as a system of equations or as simple as a geometrical figure. As the word "model" suggests, it shares characteristics of the real-world situation that it seeks to represent.

Mathematical Modelling Process



Thinking skills and heuristics

Thinking skills refers to the ability to classify, compare, analyse, identify patterns and relationships, generalise, deduce and visualise. Heuristics are general strategies that students can use to solve non-routine problems. These include using a representation (e.g. drawing a diagram, tabulating), making a guess (e.g. trial and error/ guess and check, making a supposition), walking through the process (e.g. working backwards) and changing the problem (e.g. simplifying the problem, considering special cases).

- *Metacognition*

  Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning.

- *Attitudes*

  Attitudes refer to the affective aspects of mathematics learning such as:
  - beliefs about mathematics and its usefulness;
  - interest and enjoyment in learning mathematics;
  - appreciation of the beauty and power of mathematics;

- confidence in using mathematics; and
- perseverance in solving a problem.

In the A-level mathematics curriculum, there is an emphasis on the development of mathematical processes, in particular, reasoning, communications and modelling.

Mathematics and 21st Century Competencies (21CC)

Learning mathematics (undergirded by the Mathematics Framework) supports the development of 21CC and the Desired Outcomes of Education. Students will have opportunities to experience mathematical investigation, reasoning, modelling and discourse, working individually as well as in groups, and using ICT tools where appropriate in the course of learning and doing mathematics. Through these experiences, students learn to think critically and inventively about the problems and their solutions, communicate and collaborate effectively with their peers in the course of learning, use technological tools and manage information[2]. The choice of contexts for the problems in the various syllabuses can help raise students' awareness of local and global issues around them. For example, problems set around population issues and health issues can help students understand the challenges faced by Singapore and those around the world[3]. Assessment will also play a part in encouraging students to pay attention to the 21CC. Classroom and national assessment would require students to think critically and inventively and communicate and explain their reasons effectively when they solve problems; and not just recalling formulae and procedures and performing computations.

[2] These opportunities, e.g. thinking critically and inventively, collaborating effectively with their peers are related to the Desired Outcomes of Education: A confident person, a self-directed learner, and an active contributor.
[3] These are related to the Desired Outcomes of Education: A concerned citizen.

2. CONTENT: H2 MATHEMATICS (FROM 2016)

Preamble

Mathematics is a basic and important discipline that contributes to the developments and understanding of the sciences and other disciplines. It is used by scientists, engineers, business analysts and psychologists, etc. to model, understand and solve problems in their respective fields. A good foundation in mathematics and the ability to reason mathematically are therefore essential for students to be successful in their pursuit of various disciplines.

H2 Mathematics is designed to prepare students for a range of university courses, such as mathematics, science, engineering and related courses, where a good foundation in mathematics is required. It develops mathematical thinking and reasoning skills that are essential for further learning of mathematics. Through the applications of mathematics, students also develop an appreciation of mathematics and its connections to other disciplines and to the real world.

Syllabus Aims

The aims of *H2 Mathematics* are to enable students to:

(a) acquire mathematical concepts and skills to prepare for their tertiary studies in mathematics, sciences, engineering and other related disciplines;

(b) develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving;

(c) connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines; and

(d) experience and appreciate the nature and beauty of mathematics and its value in life and other disciplines.

Content Description

There are 6 content strands in *H2 Mathematics*, namely, *Functions and Graphs*, *Sequences and Series*, *Vectors*, *Introduction to Complex Numbers*, *Calculus,* and *Probability and Statistics*.

a) <u>Functions and Graphs</u> provides a more abstract treatment of functions and their properties and studies the characteristics of a wider class of graphs including graphs defined parametrically, as well as transformation of graphs, techniques for solving equations, inequalities and system of equations.

b) <u>Sequences and Series</u> provides a useful tool for describing changes in discrete models and includes special series and sequences such as arithmetic and geometric progressions, and the concepts of convergence and infinity.

c) Vectors provides a useful tool for physical sciences as well as a means to describe and work with objects such as points, lines and planes in two- and three-dimensional spaces.

d) Complex Numbers provides an introduction to complex numbers as an extension of the number system and includes complex roots of polynomial equations, the four operations and the representation of complex numbers in exponential or polar forms.

e) Calculus provides useful tools for analysing and modelling change and behaviour and includes extension of differentiation and integration from *Additional Mathematics*, with additional techniques and applications such as power series and differential equations.

f) Probability and Statistics provides the foundation for modelling chance phenomena and making inferences with data and includes an introduction to counting techniques, computation of probability, general and specific discrete distribution models, normal distributions, sampling and hypothesis testing as well as correlation and regression.

There are many connections that can be made between the topics within each strand and across strands, even though the syllabus content are organised in strands. These connections will be emphasised so as to enable students to integrate the concepts and skills in a coherent manner to solve problems.

Knowledge of the content of *O-Level Mathematics* and part of *Additional Mathematics* is assumed in this syllabus.

Applications and Contexts

As *H2 Mathematics* is designed for students who intend to pursue further study in mathematics, science and engineering courses, students will be exposed to the applications of mathematics in sciences and engineering, so that they can appreciate the value and utility of mathematics in these likely courses of study.

The list below illustrates the kinds of contexts that the mathematics learnt in the syllabus may be applied. It is by no means exhaustive.

Applications and contexts	**Some possible topics involved**
Kinematics and dynamics (e.g. free fall, projectile motion, collisions)	Functions; Calculus; Vectors
Optimisation problems (e.g. maximising strength, minimising surface area)	Inequalities; System of linear equations; Calculus
Electrical circuits	Complex numbers; Calculus
Population growth, radioactive decay, heating and cooling problems	Differential equations
Financial Mathematics (e.g. banking,	Sequences and series; Probability; Sampling

insurance)	distributions
Standardised testing	Normal distribution; Probability
Market research (e.g. consumer preferences, product claims)	Sampling distributions; Hypothesis testing; Correlation and regression
Clinical research (e.g. correlation studies)	Sampling distributions; Hypothesis testing; Correlation and regression

While students will be exposed to applications and contexts beyond mathematics, they are not expected to learn them in depth. Students should be able to use given information to formulate and solve the problems, applying the relevant concepts and skills and interpret the solution in the context of the problem.

	Topic / Sub-topic and Content	Learning Experiences and Applications
1	**Functions and Graphs**	
1.1	Functions Include: • concepts of function, domain and range • use of notations such as $\mathrm{f}(x)=x^2+5$, $\mathrm{f}: x \mapsto x^2+5$, $\mathrm{f}^{-1}(x)$, $\mathrm{fg}(x)$ and $\mathrm{f}^2(x)$ • finding inverse functions and composite functions • conditions for the existence of inverse functions and composite functions • domain restriction to obtain an inverse function • relationship between a function and its inverse Exclude the use of the relation $(\mathrm{fg})^{-1}=\mathrm{g}^{-1}\mathrm{f}^{-1}$, and restriction of domain to obtain a composite function.	Examples of what students would do as part of their learning: (1) discuss examples and non-examples of functions, inverse functions, and composite functions in the context of sciences e.g. motion of particles; (2) examine examples of functions presented in different forms, as an algebraic expression, as a graph or as a table; (3) reason the need for domain restriction to obtain an inverse function and relate inverse trigonometric functions to the principal angles given by $\tan^{-1}$, $\sin^{-1}$, $\cos^{-1}$ in the calculator; and (4) explore and describe the graphical relationship between a function and its inverse using a graphing tool.
1.2	Graphs and transformations Include: • use of a graphing calculator to graph a given function • important characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes of the following: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$; $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ $y=\frac{ax+b}{cx+d}$ $y=\frac{ax^2+bx+c}{dx+e}$ • determining the equations of asymptotes, axes of symmetry, and restrictions on the possible values of *x* and/or *y* • effect of transformations on the graph of $y=\mathrm{f}(x)$ as represented by $y=a\mathrm{f}(x)$, $y=\mathrm{f}(x)+a$, $y=\mathrm{f}(x+a)$ and $y=\mathrm{f}(ax)$ and combinations of these transformations • relating the graphs of $y=\mathrm{f}^{-1}(x)$, $y=\lvert\mathrm{f}(x)\rvert$, $y=\mathrm{f}(\lvert x\rvert)$, and $y=\frac{1}{\mathrm{f}(x)}$ to the graph of $y=\mathrm{f}(x)$	Examples of what students would do as part of their learning: (1) investigate the characteristics of graphs using a graphing tool; (2) suggest an algebraic expression for a function based on the characteristics of a given graph; (3) interpret a composite function as a sequence of transformations in a certain order, and investigate the effect of transformations to a graph in different orders; and (4) model a projectile motion or circular motion using a pair of parametric equations.

	Topic / Sub-topic and Content	Learning Experiences and Applications
	• simple parametric equations and their graphs	
1.3	Equations and inequalities Include: • formulating an equation, a system of linear equations, or inequalities from a problem situation • solving an equation exactly or approximately using a graphing calculator • solving a system of linear equations using a graphing calculator • solving inequalities of the form $\frac{\mathrm{f}(x)}{\mathrm{g}(x)} > 0$ where $\mathrm{f}(x)$ and $\mathrm{g}(x)$ are linear expressions or quadratic expressions that are either factorisable or always positive • concept of $\lvert x \rvert$, and use of relations $\lvert x-a \rvert < b \Leftrightarrow a-b < x < a+b$ and $\lvert x-a \rvert > b \Leftrightarrow x < a-b$ or $x > a+b$, in the course of solving inequalities • solving inequalities by graphical methods	Examples of what students would do as part of their learning: (1) discuss the solution(s), or lack of it, of a system of linear equations in the context of the problem; (2) discuss the limitations of using a graphing calculator to obtain the answers to a problem; and (3) model and solve problems using a system of linear equations, and discuss the assumptions and interpret the solution in the context.
2	**Sequences and Series**	
2.1	Sequences and series Include: • concepts of sequence and series for finite and infinite cases • sequence as function $y = \mathrm{f}(n)$ where n is a positive integer • relationship between u_n (the nth term) and S_n (the sum to n terms) • sequence given by a formula for the nth term • use of $\sum$ notation • sum and difference of two series • summation of series by the method of differences • convergence of a series and the sum to infinity • formula for the nth term and the sum of a finite arithmetic series • formula for the nth term and the sum of a finite geometric series • condition for convergence of an infinite geometric series • formula for the sum to infinity of a convergent geometric series	Examples of what students would do as part of their learning: (1) relate sequence and series to functions with positive integers as the domain; (2) tell the story of how Frederick Gauss computed the sum 1+2+…+100, and formulate a proof for the sum of an AP; (3) investigate the convergence and divergence of sequences and series, including geometric series, using a spreadsheet or graphing tool; and (4) model and solve problems involving the total vertical distance covered by a bouncing ball, and simple investment or loan using a geometric series.

	Topic / Sub-topic and Content	Learning Experiences and Applications
3	**Vectors**	
3.1	Basic properties of vectors in two- and three dimensions Include: • addition and subtraction of vectors, multiplication of a vector by a scalar, and their geometrical interpretations • use of notations such as $\begin{pmatrix} x \\ y \end{pmatrix}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i}+y\mathbf{j}$, $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, $\overrightarrow{AB}$, $\mathbf{a}$ • position vectors, displacement vectors and direction vectors • magnitude of a vector • unit vectors • distance between two points • concept of direction cosines • collinearity • use of the ratio theorem in geometrical applications	Examples of what students would do as part of their learning: (1) relate vectors to displacement, velocity and acceleration; (2) relate concepts in vectors to concepts in co-ordinate geometry; (3) prove that the three medians of a triangle intersect one another at two thirds the distance from the vertex to the midpoint of the opposite side using the ratio theorem; and (4) model the projectile motion of a particle using vectors.
3.2	Scalar and vector products in vectors Include: • concepts of scalar product and vector product of vectors and their properties • calculation of the magnitude of a vector and the angle between two vectors • geometrical meanings of $\lvert\mathbf{a}\cdot\hat{\mathbf{n}}\rvert$ and $\lvert\mathbf{a}\times\hat{\mathbf{n}}\rvert$, where $\hat{\mathbf{n}}$ is a unit vector Exclude triple products $\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}$ and $\mathbf{a}\times\mathbf{b}\times\mathbf{c}$.	Examples of what students would do as part of their learning: (1) relate the magnitude of a scalar product and a vector product to the length of projection and area of a triangle or quadrilateral; (2) relate the direction of a vector product to Fleming's right-hand rule for determining the direction of current in a magnetic field; (3) relate scalar product to work done by a force pulling an object at an angle; and (4) explore the properties of scalar product between two vectors using a dynamic geometry tool.
3.3	Three-dimensional vector geometry Include: • vector and cartesian equations of lines and planes • finding the foot of the perpendicular and distance from a point to a line or to a plane • finding the angle between two lines, between a line and a plane, or between two planes • relationships between	Examples of what students would do as part of their learning: (1) relate the vector equation of a line to the concept of parametric functions; (2) examine the role of the value of the parameter(s) in a vector equation of a straight line and a plane; (3) explore and describe the geometrical relationship between: 2 lines, a line and a

	Topic / Sub-topic and Content	Learning Experiences and Applications
	(i) two lines (coplanar or skew) (ii) a line and a plane (iii) two planes Exclude: • finding the shortest distance between two skew lines • finding an equation for the common perpendicular to two skew lines	plane, and 2 planes, with the help of a dynamic geometry tool; (4) relate the solutions to a system of linear equations to the geometrical meaning of these solutions; (5) model a 3D object with plane surfaces using vector equations; (6) model and solve problems involving reflection of line rays, shadow from different light sources; and (7) read on the application of vectors in real life. (http://plus.maths.org/content/os/issue42/features/lasenby/index)
4	**Introduction to Complex Numbers**	
4.1	Complex numbers expressed in Cartesian form Include: • extension of the number system from real numbers to complex numbers • complex roots of quadratic equations • conjugate of a complex number • four operations of complex numbers • equality of complex numbers • conjugate roots of a polynomial equation with real coefficients	Examples of what students would do as part of their learning: (1) prove by contradiction that $x^2+1=0$ has no real solution, using the fact that the square of a real number is always non-negative; (2) read and discuss the article, “The 17 Equations That Changed The Course of History” (http://www.businessinsider.sg/17-equations-that-changed-the-world-2014-3/#.VD32EFe1Vel); (3) solve polynomial equations with (a) all real; (b) both real and imaginary; coefficients and discover the nature of roots of an equation; (4) prove that complex roots of a polynomial equation with real coefficients occur in conjugate pairs; and (5) relate the addition and subtraction of complex numbers to addition and subtraction of vectors.

	Topic / Sub-topic and Content	Learning Experiences and Applications
4.2	Complex numbers expressed in polar form Include: • representation of complex numbers in the Argand diagram • complex numbers expressed in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, or $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$ • calculation of modulus (r) and argument (θ) of a complex number • multiplication and division of two complex numbers expressed in polar form	Examples of what students would do as part of their learning: (1) read about Leonhard Euler and the beauty of the Euler formula; and (2) read and discuss the applications of complex numbers in various fields such as electronics and engineering (e.g. the use of fractals in RFID devices).
5	**Calculus**	
5.1	Differentiation Include: • graphical interpretation of (i) $\mathrm{f}'(x) > 0$, $\mathrm{f}'(x) = 0$ and $\mathrm{f}'(x) < 0$ (ii) $\mathrm{f}''(x) > 0$ and $\mathrm{f}''(x) < 0$ • relating the graph of $y = \mathrm{f}'(x)$ to the graph of $y = \mathrm{f}(x)$ • differentiation of simple functions defined implicitly or parametrically • determining the nature of the stationary points (local maximum and minimum points and points of inflexion) analytically, in simple cases, using the first derivative test or the second derivative test • locating maximum and minimum points using a graphing calculator • finding the approximate value of a derivative at a given point using a graphing calculator • finding equations of tangents and normals to curves, including cases where the curve is defined implicitly or parametrically • local maxima and minima problems • connected rates of change problems Exclude finding non-stationary points of inflexion and finding second derivative of functions defined parametrically.	Examples of what students would do as part of their learning: (1) derive the derivative of simple functions, e.g. x^2, $\frac{1}{x}$, $\sin x$ and e^x from first principles; (2) draw the graphs of $y = \mathrm{f}'(x)$ and $y = \mathrm{f}(x)$ using a graphing tool and describe the features of one graph with reference to the other; (3) deduce the distance-time graph from a speed-time graph, or the speed-time graph from an acceleration-time graph; (4) model and solve problems involving rate of change e.g. kinematics, radioactive decay, population growth, charging of capacitors, heating and cooling; (5) model and solve problems involving optimisation or equilibrium points; and (6) find the direction of a projectile at a given time, where the horizontal and vertical distances are given in terms of time.

	Topic / Sub-topic and Content	Learning Experiences and Applications
5.2	Maclaurin series Include: • standard series expansion of $(1+x)^n$ for any rational n, $e^x, \sin x, \cos x$ and $\ln(1+x)$ • derivation of the first few terms of the Maclaurin series by ○ repeated differentiation, e.g. $\sec x$ ○ repeated implicit differentiation, e.g. $y^3+y^2+y=x^2-2x$ ○ using standard series, e.g. $e^x \cos 2x$, $\ln\left(\frac{1+x}{1-x}\right)$ • range of values of x for which a standard series converges • concept of "approximation" • small angle approximations: $\sin x \approx x$, $\cos x \approx 1-\frac{1}{2}x^2$, $\tan x \approx x$ Exclude derivation of the general term of the series.	Examples of what students would do as part of their learning: (1) observe and compare the region where the graphs of a function and its standard series expansion are close to each other using a graphing tool by varying the number of terms in the standard series expansion of the function; and (2) deduce an approximation for the *n*th power of a number close to 1 without using a calculator e.g. $(1.004)^{12}$.
5.3	Integration techniques Include: • integration of $f'(x)[f(x)]^n$ (including $n=-1$), $f'(x)e^{f(x)}$ $\sin^2 x, \cos^2 x, \tan^2 x$, $\sin mx \cos nx, \cos mx \cos nx$ and $\sin mx \sin nx$ $\frac{1}{a^2+x^2}, \frac{1}{\sqrt{a^2-x^2}}, \frac{1}{a^2-x^2}$ and $\frac{1}{x^2-a^2}$ • integration by a given substitution • integration by parts	Examples of what students would do as part of their learning: (1) relate integration by substitution to chain rule for differentiation; (2) discuss why a given substitution works and suggest alternative substitutions; and (3) deduce the formula for integration by parts from the product rule for differentiation.

	Topic / Sub-topic and Content	Learning Experiences and Applications
5.4	Definite integrals Include: • concept of definite integral as a limit of sum • definite integral as the area under a curve • evaluation of definite integrals • finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves • area below the x-axis • finding the area under a curve defined parametrically • finding the volume of revolution about the x- or y-axis • finding the approximate value of a definite integral using a graphing calculator Exclude finding the volume of revolution about the x-axis or y-axis where curve is defined parametrically.	Examples of what students would do as part of their learning: (1) verify that the area under a graph is given by the limit of an infinite sum of rectangular strips using a spreadsheet or program; (2) visualise how a solid (e.g. cone or cylinder) can be generated from rotation of an area under a curve about the x-axis or y-axis; (3) estimate the area and volume of real-world objects (e.g. area of a wall of irregular shape; volume of a bowl); (4) discuss how the volume of revolution could be found when the axis of revolution is a horizontal or vertical line other than the axes; (5) model and solve problems related to motion of particles, work done or turning force; and (6) model and solve problems related to the centre of mass of a lamina object.
5.5	Differential equations Include: • solving for the general solutions and particular solutions of differential equations of the forms (i) $\frac{dy}{dx} = f(x)$ (ii) $\frac{dy}{dx} = f(y)$ (iii) $\frac{d^2y}{dx^2} = f(x)$ including those that can be reduced to (i) and (ii) by means of a given substitution • formulating a differential equation from a problem situation • interpreting a differential equation and its solution in terms of a problem situation	Examples of what students would do as part of their learning: (1) model and solve problems related to radioactive decay, heating and cooling of substances, charging of capacitors, population growth, spread of diseases, chemical reaction; and (2) model and solve problems related to motion of a particle with resistance.
6	**Probability and Statistics**	
6.1	Probability Include: • addition and multiplication principles for counting • concepts of permutation (^nP_r) and combination (^nC_r) • arrangements of objects in a line or in a circle,	Examples of what students would do as part of their learning: (1) discuss and generate examples of problems where the addition and multiplication principles are applied; (2) explain the relationship between ^nP_r and ^nC_r;

<table>
<tr><th></th><th>Topic / Sub-topic and Content</th><th>Learning Experiences and Applications</th></tr>
<tr><td></td><td>including cases involving repetition and restriction
• addition and multiplication of probabilities
• mutually exclusive events and independent events
• use of tables of outcomes, Venn diagrams, tree diagrams, and permutations and combinations techniques to calculate probabilities
• calculation of conditional probabilities in simple cases
• use of:
$\mathrm{P}(A') = 1 - \mathrm{P}(A)$
$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$
$\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$</td><td>(3) discuss real-life examples where probability is required;
(4) discuss the role of conditional probability in the Monty Hall problem, prosecutors’ fallacy or Sally Clark case;
(5) compare between mutually exclusive events and independent events;
(6) verify that the relative frequency converges to the theoretical probability by simulating the tossing of a fair coin using a random number generator; and
(7) model and solve problems involving games of chance, human behaviours, risks or propensity.</td></tr>
<tr><td>6.2</td><td>Discrete random variables

Include:
• concept of discrete random variables, probability distributions, expectations and variances
• concept of binomial distribution $\mathrm{B}(n, p)$ as an example of a discrete probability distribution and use of $\mathrm{B}(n, p)$ as a probability model, including conditions under which the binomial distribution is a suitable model
• use of mean and variance of binomial distribution (without proof)

Exclude finding cumulative distribution function of a discrete random variable.</td><td>Examples of what students would do as part of their learning:

(1) give examples of real-world situations that involve discrete random variables;
(2) explain and relate the expectation and variance of a random variable to the mean and standard deviation of a frequency distribution;
(3) give examples of real-world situations that can be modelled using a binomial distribution;
(4) explain why a given real-world situation cannot be modelled using a binomial distribution;
(5) relate the expansion of $(p+q)^n$, where $p+q=1$ and $n \in \mathbb{Z}^+$ to the probability distribution function for a binomial random variable; and
(6) explore the effects of varying n and p on the shape of the binomial distribution using a graphing tool.</td></tr>
<tr><td>6.3</td><td>Normal distribution

Include:
• concept of a normal distribution as an example of a continuous probability model and its mean and variance; use of $\mathrm{N}(\mu, \sigma^2)$ as a probability model
• standard normal distribution
• finding the value of $\mathrm{P}(X < x_1)$ or a related</td><td>Examples of what students would do as part of their learning:

(1) compare and approximate the normal and binomial distributions for large n with np being constant using a statistical tool;
(2) suggest ways to compare two scores from different tests with the test scores following normal distributions;
(3) explore the characteristics of a normal</td></tr>
</table>

	Topic / Sub-topic and Content	Learning Experiences and Applications
	probability, given the values of x_1, μ, σ • symmetry of the normal curve and its properties • finding a relationship between x_1, μ, σ given the value of $\mathrm{P}(X < x_1)$ or a related probability • solving problems involving the use of $\mathrm{E}(aX+b)$ and $\mathrm{Var}(aX+b)$ • solving problems involving the use of $\mathrm{E}(aX+bY)$ and $\mathrm{Var}(aX+bY)$, where X and Y are independent Exclude normal approximation to binomial distribution.	curve: shape, centre, spread, and probability as area under the curve using a graphing tool; and (4) model and solve problems involving social phenomena, quality control and standards.
6.4	Sampling Include: • concepts of population, random and non-random samples • concept of the sample mean $\overline{X}$ as a random variable with $\mathrm{E}(\overline{X}) = \mu$ and $\mathrm{Var}(\overline{X}) = \frac{\sigma^2}{n}$ • distribution of sample means from a normal population • use of the Central Limit Theorem to treat sample means as having normal distribution when the sample size is sufficiently large • calculation and use of unbiased estimates of the population mean and variance from a sample, including cases where the data are given in summarised form $\sum x$ and $\sum x^2$, or $\sum(x-a)$ and $\sum(x-a)^2$	Examples of what students would do as part of their learning: (1) discuss ways to perform a random selection from a population; (2) discuss the underlying sampling methods in reports e.g. healthcare and exit polls, and evaluate whether it is random or not; (3) simulate sampling from a population and study the distribution of the sample mean as a random variable using a statistical tool; (4) discuss the advantages of summarising the data in coded form and explain how this affects the mean and standard deviation; and (5) simulate large sampling from a population and verify that the distribution of the sample mean has a bell-shaped distribution (CLT).
6.5	Hypothesis testing Include: • concepts of null hypothesis (H_0) and alternative hypotheses (H_1), test statistic, critical region, critical value, level of significance, and *p*-value • formulation of hypotheses and testing for a population mean based on: - a sample from a normal population of known variance - a large sample from any population • 1-tail and 2-tail tests • Interpretation of the results of a hypothesis test in the context of the problem	Examples of what students would do as part of their learning: (1) suggest ways to verify and dispute a claim based on data before introducing the concept of hypothesis testing; (2) relate the notion of null and alternative hypothesis to the statement "innocent until proven guilty"; (3) identify real-world situations where hypothesis testing is evident; (4) explain the meaning and relationship between *p*-value and *Z*-value; (5) relate the error of hypothesis testing to the notion of "false positive" and "false

	Topic / Sub-topic and Content	Learning Experiences and Applications
	Exclude the use of the term 'Type I error', concept of Type II error and testing the difference between two population means.	negative" in health screening; and (6) solve problems involving healthcare, product testing, consumer preferences, lifestyle choices and quality control.
6.6	Correlation and Linear regression Include: • use of scatter diagram to determine if there is a plausible linear relationship between the two variables • correlation coefficient as a measure of the fit of a linear model to the scatter diagram • finding and interpreting the product moment correlation coefficient (in particular, values close to -1, 0 and 1) • concepts of linear regression and method of least squares to find the equation of the regression line • concepts of interpolation and extrapolation • use of the appropriate regression line to make prediction or estimate a value in practical situations, including explaining how well the situation is modelled by the linear regression model • use of a square, reciprocal or logarithmic transformation to achieve linearity Exclude: • derivation of formulae • relationship $r^2 = b_1 b_2$, where b_1 and b_2 are regression coefficients • hypothesis tests	Examples of what students would do as part of their learning: (1) suggest ways to fit a straight line to a set of data that look linearly related with a straight line; (2) suggest ways to measure how good the straight line fits a set of data; (3) draw the least squares line for the set of 9 points $(0,0)$, $(0,1)$, $(0,-1)$, $(1,0)$, $(-1,0)$, $(1,1)$, $(1,-1)$, $(-1,1)$, $(-1,-1)$, without computing the line; (4) discuss the difference between correlation and causation; (5) contrast interpolation and extrapolation and discuss the risk of extrapolation though real-life examples; (6) investigate the relationship between the two regression lines (y-on-x and x-on-y); and (7) apply the least squares method to find the best fit line in science experiments (e.g. period versus length of pendulum, lens equation, cooling experiments) and compare that with the equation 'fit by eye'.

3. PEDAGOGY

Principles of Teaching and Learning

The following principles guide the teaching and learning of mathematics.

- Principle 1: Teaching is for learning; learning is for understanding; understanding is for reasoning and applying and, ultimately problem solving.

- Principle 2: Teaching should build on the pre-requisite knowledge for the topics; take cognisance of students' interests and experiences; and engage them in active and reflective learning.

- Principle 3: Teaching should connect learning to the real world, harness technology and emphasise 21st century competencies.

These principles capture the importance of deep and purposeful learning, student-centric pedagogies and self-directed learning, relevance to the real world, learning with technology and future orientation towards learning.

Learning Experiences

Learning mathematics is more than just learning concepts and skills. Equally important are the cognitive and metacognitive process skills. These processes are learned through carefully constructed learning experiences. The learning experiences stated in Section 2 of the syllabus link the learning of content to the development of mathematical processes. They are examples of what students would do as part of their learning. These learning experiences create opportunities for students to:

a) Engage in mathematical discussion where students actively reason and communicate their understanding to their peers and solve problems collaboratively;

b) Construct mathematical concepts (e.g. to develop their own measure of linear relationship before being taught the formal concept) and form their own understanding of the concepts;

c) Model and apply mathematics to a range of real-world problems (e.g. using exponential growth model to model population growth) afforded by the concepts and models in the syllabus;

d) Make connections between ideas in different topics and between the abstract mathematics and the real-world applications and examples; and

e) Use ICT tools to investigate, form conjecture and explore mathematical concepts (e.g. properties of graphs and their relationship with the algebraic expressions that describe the graph).

The learning experiences also contribute to the development of 21CC. For example, to encourage students to be inquisitive, the learning experiences include opportunities where students discover mathematical results on their own. To support the development of collaborative and communication skills, students are given opportunities to work together on a problem and present their ideas using appropriate mathematical language and methods. To develop habits of self-directed learning, students are given opportunities to set learning goals and work towards them purposefully.

Teaching and Learning Approaches

To better cater to the learning needs of JC students and to equip them with 21CC, students would experience a blend of pedagogies. Pedagogies that are constructivist in nature complement direct instruction. A constructivist classroom features greater student participation, collaboration and discussion, and greater dialogue between teachers and peers. Students take on a more active role in learning, and construct new understandings and knowledge. The teacher's role is to facilitate the learning process (e.g. through more in-depth dialogue and questioning) and guide students to build on their prior knowledge, and provide them with opportunities for more ownership and active engagement during learning.

Below are examples of possible strategies that support the constructivist approach to learning:

- Activity based learning e.g. individual or group work, problem solving
- Teacher-directed inquiry e.g. demonstration, posing questions
- Flipped classroom e.g. independent study, followed by class discussion
- Seminar e.g. mathematical discussion and discourse
- Case studies e.g. reading articles, analysing real data
- Project e.g. mathematical modelling, statistical investigation
- Lab work e.g. simulation, investigation using software and application

4. ASSESSMENT

Role of Assessment

The role of assessment is to improve teaching and learning. For students, assessment provides them with information about how well they have learned and how they can improve. For teachers, assessment provides them with information about their students' learning and how they can adjust their instruction. Assessment is therefore an integral part of the interactive process of teaching and learning.

Assessment in mathematics should focus on students':

- understanding of mathematics concepts (going beyond simple recall of facts);
- ability to draw connections and integrate ideas across topics;
- capacity for logical thought, particularly, the ability to reason, communicate, and interpret; and
- ability to formulate, represent and solve problems within mathematics and other contexts.

The purpose of assessments can be broadly classified as summative, formative, and diagnostic.

- Summative assessments, such as tests and examinations, measure what students have learned. Students will receive a score or a grade.
- Formative and diagnostic assessments are used to support learning and to provide timely feedback for students on their learning, and to teachers on their teaching.

Classroom Assessments

Assessments in the mathematics classroom are primarily formative and diagnostic in purpose. Classroom assessments include the questions teachers asked during lessons, the homework assigned to students, and class tests given at different times of the academic year. For these assessments to be formative, feedback to students is important. Students should use the feedback from these assessments to understand where they are in their learning and how to improve their learning.

GCE A-Level National Examination

Students will take the national examination in their final year. The national examination is a summative assessment that measures the level of attainment of the outcomes stated in the syllabuses.

The national examination code for the paper is **9758**. The examination syllabus can be found in the SEAB website. Important information about the examination is reproduced here.

Assessment Objectives (AO)

The assessment will test students' abilities to:

AO1 Understand and apply mathematical concepts and skills in a variety of problems, including those that may be set in unfamiliar contexts, or require integration of concepts and skills from more than one topic.

AO2 Formulate real-world problems mathematically, solve the mathematical problems, interpret and evaluate the mathematical solutions in the context of the problems.

AO3 Reason and communicate mathematically through making deductions and writing mathematical explanations, arguments and proofs.

The examinations will be based on the topic/sub-topic and content list on page 9 – 18. Knowledge of *O-level Mathematics* and part of *Additional Mathematics* is assumed.

Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics.

While problems may be set based in contexts, no assumptions will be made about the knowledge of the contexts. All information will be self-contained within the problem.

The assumed knowledge for *O-level Additional Mathematics* is stated below.

Content from O Level Additional Mathematics	
ALGEBRA	
A1	Equations and inequalities • conditions for a quadratic equation to have: (i) two real roots (ii) two equal roots (iii) no real roots • conditions for ax^2+bx+c to be always positive (or always negative) • solving simultaneous equations with at least one linear equation, by substitution
A2	Indices and surds • four operations on indices and surds • rationalising the denominator
A3	Polynomials and partial fractions • multiplication and division of polynomials • use of remainder and factor theorems • partial fractions with cases where the denominator is not more complicated than: - $(ax + b)(cx + d)$ - $(ax + b)(cx + d)^2$ - $(ax + b)(x^2 + c^2)$
A4	Power, Exponential, Logarithmic, and Modulus functions • power functions $y = ax^n$, where n is a simple rational number, and their

Content from O Level Additional Mathematics	
	graphs • functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs • laws of logarithms • equivalence of $y = a^x$ and $x = \log_a y$ • change of base of logarithms • function $\lvert x \rvert$ and graph of $\lvert f(x) \rvert$, where $f(x)$ is linear, quadratic or trigonometric • solving simple equations involving exponential and logarithmic functions
GEOMETRY AND TRIGONOMETRY	
B5	Coordinate geometry in two dimensions • graphs of equations $y^2 = kx$ • coordinate geometry of the circle with the equation in the form $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$
B6	Trigonometric functions, identities and equations • six trigonometric functions, and principal values of the inverses of sine, cosine and tangent • trigonometric equations and identities (see List of Formulae) • expression of $a\cos\theta + b\sin\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$
CALCULUS	
C7	Differentiation and integration • derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point • derivative as rate of change • derivatives of x^n for any rational n, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$, together with constant multiples, sums and differences • derivatives of composite functions • derivatives of products and quotients of functions • increasing and decreasing functions • stationary points (maximum and minimum turning points and points of inflexion) • use of second derivative test to discriminate between maxima and minima • connected rates of change • maxima and minima problems • integration as the reverse of differentiation • integration of x^n for any rational n, e^x, $\sin x$, $\cos x$, $\sec^2 x$ and their constant multiples, sums and differences • integration of $(ax+b)^n$ for any rational n, $\sin(ax+b)$, $\cos(ax+b)$ and e^{ax+b}

Scheme of Examination Papers

There will be two 3-hour papers, each carrying 50% of the total mark, and each marked out of 100, as follows:

PAPER 1 (3 hours)
A paper consisting of 10 to 12 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

There will be at least two questions on application of Mathematics in real-world contexts, including those from sciences and engineering. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Students will be expected to answer **all** questions.

PAPER 2 (3 hours)
A paper consisting of 2 sections, Sections A and B.

Section A (Pure Mathematics – 40 marks) will consist of 4 to 5 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.
Section B (Probability and Statistics – 60 marks) will consist of 6 to 8 questions of different lengths and marks based on the Probability and Statistics section of the syllabus.

There will be at least two questions in Section B on application of Mathematics in real-world contexts, including those from sciences and engineering. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Students will be expected to answer **all** questions.

Use of a graphing calculator (GC)

The use of an approved GC *without* computer algebra system will be expected. The examination papers will be set with the assumption that students will have access to GC. As a general rule, unsupported answers obtained from GC are allowed unless the question states otherwise. Where unsupported answers from GC are not allowed, students are required to present the mathematical steps using mathematical notations and not calculator commands. For questions where graphs are used to find a solution, students should sketch these graphs as part of their answers. Incorrect answers without working will receive no marks. However, if there is written evidence of using GC correctly, method marks may be awarded.

Students should be aware that there are limitations inherent in GC. For example, answers obtained by tracing along a graph to find roots of an equation may not produce the required accuracy.

List of formulae and statistical tables

Students will be provided in the examination with a list of formulae and statistical tables.

Mathematical notation

A list of mathematical notation is available at the SEAB website.

5. USEFUL REFERENCE BOOKS

Pure Mathematics	- Blank, B. E. & Krantz, S. G. (2011). Calculus: Single variable. (4th Ed.). Wiley. - Smith, R. T. & Minton, R. (2011). Calculus: Early transcendental functions. (4th Ed.). McGraw-Hill Education. - Stewart, J. (2015). Calculus: Early transcendentals. (8th Ed.). Cengage Learning. - Zill, D. G. (2013). A first course in differential equations with modelling applications. (10th Ed.). Brooks Cole.
Probability and Statistics	- Agresti, A. & Franklin, C. A. (2012). Statistics: The art and science of learning from data. (3rd Ed.). Pearson. - Crawshaw, J. & Chambers, J. (2001). A concise course in advanced level statistics. (4th Ed.). Nelson Thornes Ltd. - Dobbs, S. & Miller, J. (2003). Statistics 1, 2. Cambridge University Press. - Freedman, D., Pisani, R. & Purves, R. (2007). Statistics. (4th Ed.). W. W. Norton & Company. - McClave, J. T. & Sincich, T. T. (2012). Statistics. (12th Ed.). Pearson. - Walpole, R. E., Myers, R. H., Myers, S. L. & Ye, K. (2012). Probability & statistics for engineers & scientists. (9th Ed.). Pearson.

*The list is by no means exhaustive as they provide some samples that students can refer to. There are other reference books that students can use as well.