MATHEMATICS SYLLABUS

Pre-University

H2 Further Mathematics

Implementation starting with
2016 Pre-University One Cohort

Ministry of Education
SINGAPORE

CONTENTS

1. INTRODUCTION

Importance of Mathematics

Mathematics contributes to the developments and understanding in many disciplines. It is used extensively to model the real world, create new products and services and support data-driven decisions. A good foundation in mathematics and a keen appreciation of its potential give one a competitive edge over others.

Discipline of Mathematics

Mathematics is a study about quantities, space, patterns, relationships, chance and abstractions. Mathematical knowledge is established through rigorous proofs, derived from axioms and definitions through logical argument and reasoning. Mathematical statements or claims should be challenged and remain as conjectures until they are proven to be true.

Mathematics can be seen as a language. It is used to express, communicate and share ideas, within the scientific communities as well as with the general public. It has its own set of notations, symbols, and terminologies. It is a language that strives to be precise and concise.

The applications of mathematics transcend its own boundary, into the daily life, the real world and other disciplines. It is more than just computations. Mathematics is a powerful tool to model real world phenomena. But it has its limitations, as often mathematical models cannot capture all the complexities of real world.

Learning of Mathematics

The learning of mathematics should honour the nature of the discipline and its practices. Students should therefore learn to justify their solutions, give reasons to support their conclusions and prove mathematical statements. They should also learn to communicate mathematically, construct and discuss mathematical statements, and use the language of mathematics to develop and follow a logical chain of reasoning. In applying mathematics to solve real world problems, they should learn to formulate models, be aware of the limitations of these models and exercise care in the interpretation of mathematics solutions. Such learning experiences will provide students a glimpse of what being a mathematician is like and what mathematics is about.

Mathematics at the A-Level

In Singapore, mathematics education at the A-level plays an important role in laying the foundation for building a pool of highly skilled and analytical workforce, especially in STEM-related areas. From the period of rapid industrialisation in the 80’s to the current day of knowledge intensive industries, it continues to be highly valued by stakeholders and students preparing for tertiary education. Although mathematics is an optional subject at the A-level, it is offered by nearly all students.

The purpose of learning mathematics at the A-level is two-fold. Firstly, it provides students, regardless of the intended course of study at the university, with a useful set of tools and problem solving skills to support their tertiary study. Secondly, learning mathematics exposes students to a way of thinking that complements the ways of thinking developed through other disciplines. This contributes to the development of a well-rounded individual who is able to think deeply, broadly and differently about problems and issues.

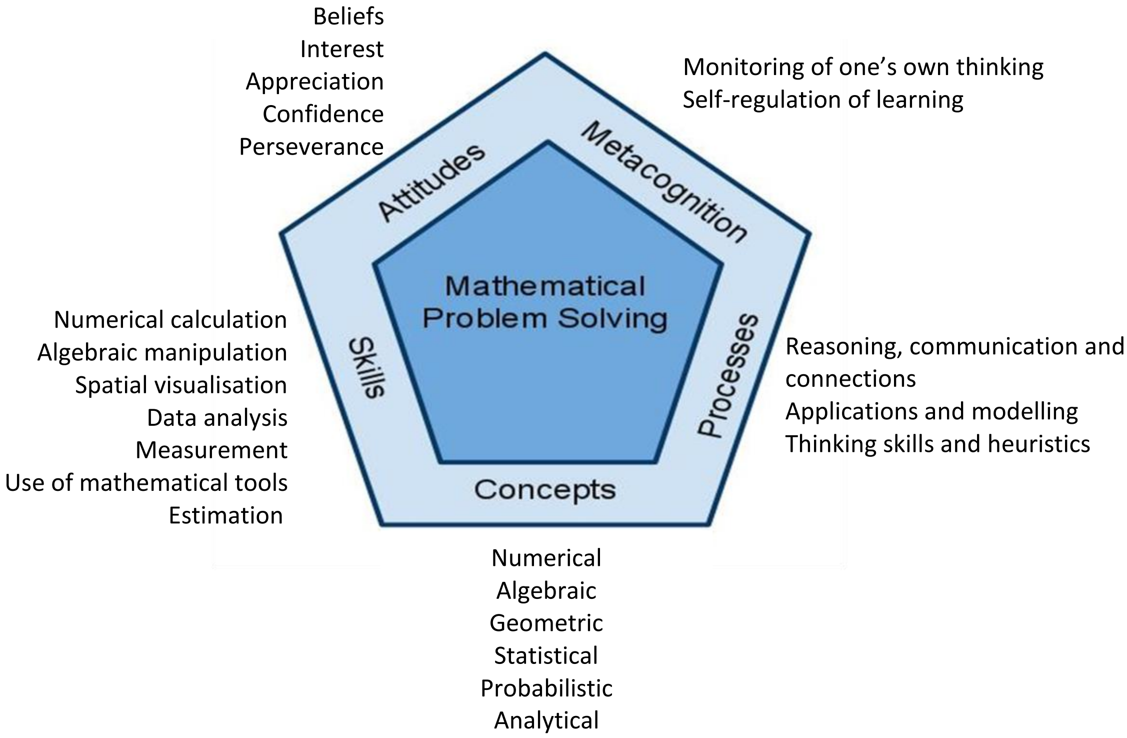
A suite of syllabuses is available to students at the A-level. The syllabuses are:

- H1 Mathematics;
- H2 Mathematics;
- H2 Further Mathematics; and
- H3 Mathematics.

The suite of syllabuses is designed for different profiles of students, to provide them with options to learn mathematics at different levels, and to varying breadth, depth or specialisation so as to support their progression to their desired choice of university courses.

Mathematics Framework

The Mathematics Framework sets the direction for curriculum and provides guidance in the teaching, learning, and assessment of mathematics. The central focus is mathematical problem solving, that is, using mathematics to solve problems. The curriculum stresses *conceptual understanding*, *skills proficiency* and *mathematical processes*, and gives due emphasis to *attitudes* and *metacognition*. These five components are inter-related.



- *Concepts*

At the A-level, students continue to study concepts and skills in the major strands of mathematics, which provide the building blocks for the learning of advanced mathematics, with varying breadth and depth depending on the syllabuses. These major strands include Algebra, Calculus, Vectors, and Probability and Statistics, which are rich in applications within mathematics and in other disciplines and the real world. These content categories are connected and interdependent.

- *Skills*

Mathematical skills refer to *numerical calculation*, *algebraic manipulation*, *spatial visualisation*, *data analysis*, *measurement*, *use of mathematical tools*, and *estimation*. The skills are specific to mathematics and are important in the learning and application of mathematics. In today's classroom, these skills also include the abilities to use spreadsheets and other software to learn and do mathematics.

- *Processes*

Mathematical processes refer to the skills involved in acquiring and applying mathematical knowledge. These include *reasoning, communication and connections*, *applications and modelling*, and *thinking skills and heuristics* that are important in mathematics.

Reasoning, communication and connections

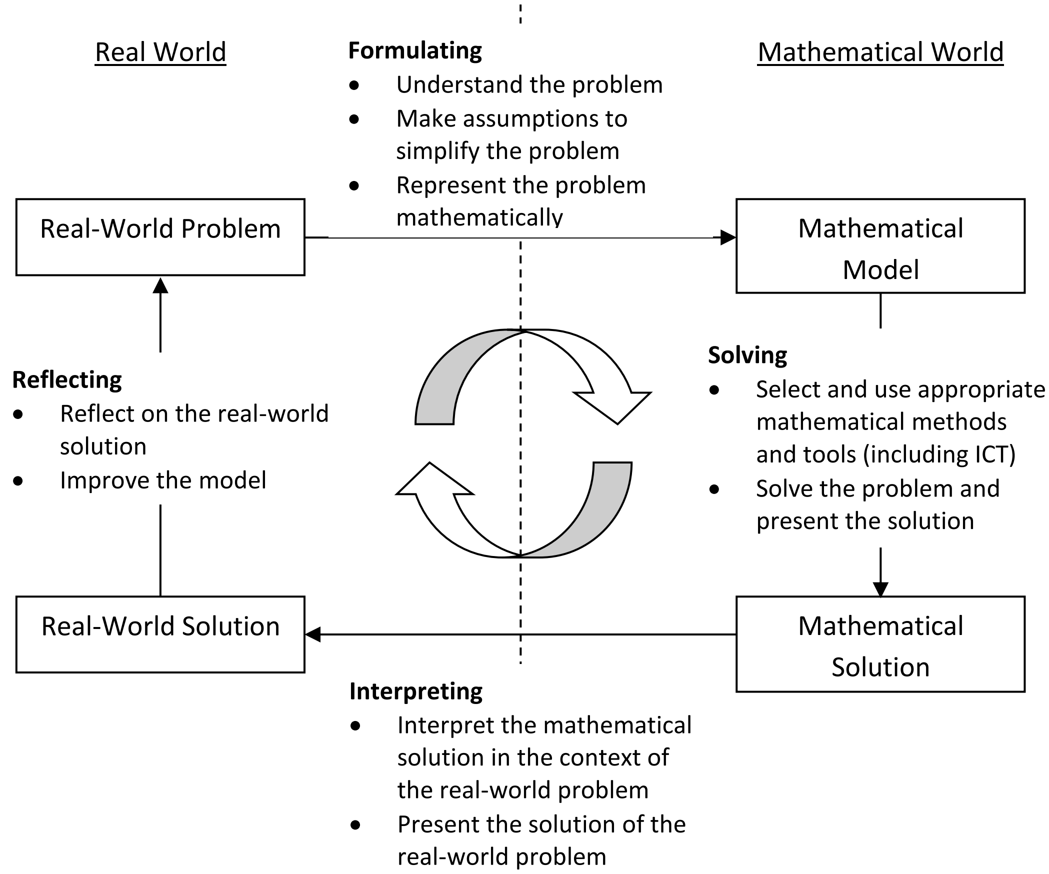
- Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments.
- Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically.
- Connections refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and the real world.

Applications and modelling

Exposing students to applications and modelling enhances their understanding and appreciation of mathematics. Mathematical modelling is the process of formulating and improving a mathematical model[1] to represent and solve real-world problems. Through mathematical modelling, students learn to deal with complexity and ambiguity by simplifying and making reasonable assumptions, select and apply appropriate mathematical concepts and skills that are relevant to the problems, and interpret and evaluate the solutions in the context of the real-world problem.

[1] A mathematical model is a mathematical representation or idealisation of a real-world situation. It can be as complicated as a system of equations or as simple as a geometrical figure. As the word "model" suggests, it shares characteristics of the real-world situation that it seeks to represent.

Mathematical Modelling Process



Thinking skills and heuristics

Thinking skills refers to the ability to classify, compare, analyse, identify patterns and relationships, generalise, deduce and visualise. Heuristics are general strategies that students can use to solve non-routine problems. These include using a representation (e.g. drawing a diagram, tabulating), making a guess (e.g. trial and error/ guess and check, making a supposition), walking through the process (e.g. working backwards) and changing the problem (e.g. simplifying the problem, considering special cases).

- *Metacognition*

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning.

- *Attitudes*

Attitudes refer to the affective aspects of mathematics learning such as:

- beliefs about mathematics and its usefulness;
- interest and enjoyment in learning mathematics;
- appreciation of the beauty and power of mathematics;

- confidence in using mathematics; and
- perseverance in solving a problem.

In the A-level mathematics curriculum, there is an emphasis on the development of mathematical processes, in particular, reasoning, communications and modelling.

Mathematics and 21st Century Competencies (21CC)

Learning mathematics (undergirded by the Mathematics Framework) supports the development of 21CC and the Desired Outcomes of Education. Students will have opportunities to experience mathematical investigation, reasoning, modelling and discourse, working individually as well as in groups, and using ICT tools where appropriate in the course of learning and doing mathematics. Through these experiences, students learn to think critically and inventively about the problems and their solutions, communicate and collaborate effectively with their peers in the course of learning, use technological tools and manage information[2]. The choice of contexts for the problems in the various syllabuses can help raise students' awareness of local and global issues around them. For example, problems set around population issues and health issues can help students understand the challenges faced by Singapore and those around the world[3]. Assessment will also play a part in encouraging students to pay attention to the 21CC. Classroom and national assessment would require students to think critically and inventively and communicate and explain their reasons effectively when they solve problems; and not just recalling formulae and procedures and performing computations.

[2] These opportunities, e.g. thinking critically and inventively, collaborating effectively with their peers are related to the Desired Outcomes of Education: A confident person, a self-directed learner, and an active contributor.
[3] These are related to the Desired Outcomes of Education: A concerned citizen.

2. CONTENT: H2 FURTHER MATHEMATICS (FROM 2016)

Preamble

Mathematics drives many of the advancements in science, engineering, economics and technology. It is at the heart of many of the innovative products and services today. A strong grounding in mathematics is essential for students who aspire to be scientists, engineers or any other professionals who require mathematical tools to solve complex problems.

H2 Further Mathematics is designed for students who are mathematically-inclined and who intend to specialise in mathematics, science, engineering or disciplines with higher demand on mathematical skills. It extends and expands on the range of mathematics and statistics topics in *H2 Mathematics* and provides these students with a head start in learning a wider range of mathematical methods and tools that are useful for solving more complex problems in mathematics and statistics.

***H2 Further Mathematics* is to be offered with *H2 Mathematics* as a double mathematics course.**

Syllabus Aims

The aims of *H2 Further Mathematics* are to enable students to:

(a) acquire a wider range of mathematical concepts and stronger set of mathematical skills for their tertiary studies in mathematics, sciences, engineering and other related disciplines with a heavier demand on mathematics;

(b) develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving;

(c) connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines; and

(d) experience and appreciate the rigour and abstraction in the discipline.

Content Description

H2 Further Mathematics comprises 3 content strands, namely, *Algebra and Calculus*, *Discrete Mathematics, Matrices and Numerical Methods*, and *Probability and Statistics*.

a) Algebra and Calculus plays a central role in the understanding, development and applications of many branches of mathematics. The strand adds breadth and depth to the topics taught in *H2 Mathematics* by broadening and deepening the understanding of important mathematical concepts and opening up a wider range of applications that may be useful for the students. It will include mathematical induction, polar curves, conic sections and additional topics in complex numbers and calculus. Through these topics, students will be exposed to a wider range of applications in science and engineering, and develop stronger reasoning skills through the writing of mathematical proof.

b) Discrete Mathematics focuses on discrete structures that have many modern real-world applications, especially in computing. Numerical Methods provide useful tools and algorithms to solve problems where exact solutions are not available. This strand adds breadth by introducing problems of discrete nature, in addition to the continuous ones that require calculus, and an 'algorithmic approach' to problem solving in addition to the analytic or algebraic approach that could expose students to basic programming. It will include the study of recurrence relations, matrices and linear spaces and algorithms to solve problems.

c) Probability and Statistics provides the concepts, skills and models to study phenomena where randomness, chance and uncertainty are present. This strand adds breadth and depth to the topics taught in *H2 Mathematics* by broadening and deepening the understanding of important probability and statistical concepts and offering a larger statistical toolkit that may be useful for future studies and research work. The topics include more statistical and probability models (e.g. general and special continuous random variables such as exponential distribution, additional discrete probability model such as Poisson) and a wider range of applications and statistical methods (e.g. paired sampled tests, non-parametric tests, chi-squared tests) that will be useful in areas as far ranging as genetics and politics.

There are many connections that can be made between the topics within each strand and across strands, even though the syllabus content are organised in strands. These connections will be emphasised so as to enable students to integrate the concepts and skills in a coherent manner to solve problems.

Knowledge of the content of *H2 Mathematics* is assumed.

Applications and Contexts

As *H2 Further Mathematics* is designed for students who intend to specialise in mathematics, science, engineering or disciplines with higher demand on mathematical skills, students will be exposed to the applications of mathematics in the sciences and engineering, so that they can appreciate the value and utility of mathematics in these likely courses of study.

The list below illustrates the kinds of contexts that the mathematics learnt in the syllabus may be applied. It is by no means exhaustive.

Applications and contexts	**Some possible topics involved**
Kinematics and dynamics (e.g. free fall, projectile motion, orbital motion, collisions)	Functions; Calculus; Vectors
Movie graphics	Vectors
Optics (design of mirrors)	Functions; Conic Sections
Optimisation problems (e.g. maximising strength, minimising surface area)	Inequalities; System of linear equations; Calculus
Electrical circuits (including alternating current circuit)	Complex numbers; Calculus

Population growth (e.g. spread of diseases), radioactive decay, heating and cooling problems, mixing, chemical changes, charging	Differential equations
Search engines, cryptography, digital music	Matrices and linear spaces
Financial Mathematics (e.g. banking, insurance)	Sequences and series; Probability; Sampling distributions
Standardised testing	Normal distribution; Probability
Market research (e.g. consumer preferences, product claims)	Sampling distributions; Hypothesis testing; Correlation and regression
Clinical research (e.g. correlation studies)	Sampling distributions; Hypothesis testing; Correlation and regression
Polling	Confidence intervals; Hypothesis testing
Genetics	Chi-square tests

While students will be exposed to applications and contexts beyond mathematics, they are not expected to learn them in depth. Students should be able to use given information to formulate and solve the problems, applying the relevant concepts and skills and interpret the solution in the context of the problem.

	Topic / Sub-topic and Content	Learning Experiences and Applications
1	**Algebra and Calculus**	
1.1	Mathematical induction • Use of method of mathematical induction to establish a given result involving series and recurrence relations, derivatives, inequalities, or divisibility • Formulation of conjectures	Examples of what students would do as part of their learning: (1) identify features of mathematical statements that can be proved by induction; (2) explain the logic of mathematical induction; (3) relate induction to the idea of domino effect; and (4) critique and correct erroneous 'proofs'.
1.2	Complex numbers • Geometrical effects of conjugation, addition, subtraction, multiplication and division of complex numbers • Loci of simple equations and inequalities such as $\|z-c\|\le r$, $\|z-a\|=\|z-b\|$ and $\arg(z-a)=\alpha$ (excluding loci of $\|z-a\|=k\|z-b\|$, where $k\neq 1$ and $\arg(z-a)-\arg(z-b)=\alpha$) • Use of de Moivre's theorem to find the powers and *n*th roots of a complex number, and to derive trigonometric identities	Examples of what students would do as part of their learning: (1) relate the conjugation, addition and subtraction of complex numbers to that of reflection, addition and subtraction of vectors respectively; (2) discover the geometrical effect of multiplying/dividing two complex numbers using a dynamic geometry tool; (3) explore the application of complex numbers e.g. in electrical circuits, fractals generation; (4) discuss the efficacy of using complex numbers to prove trigonometric identities; and (5) read about the use of complex numbers in the CORDIC algorithm used in calculators.
1.3	Polar curves and conic sections • Simple polar curves ($0\le\theta<2\pi$ or $-\pi<\theta\le\pi$ or a subset of either of these intervals) • Definitions and defining geometrical properties of conic sections, including their general equations: Circle $(x-h)^2+(y-k)^2=r^2$ Ellipse $\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$ Parabola $(x-h)^2=4p(y-k),\ p\neq 0;$ $(y-k)^2=4p(x-h),\ p\neq 0$ Hyperbola $\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1;$	Examples of what students would do as part of their learning: (1) discuss essential characteristics of a co-ordinate system to locate points on a plane; (2) propose alternative methods (other than the Cartesian coordinates) of locating a point before being introduced to polar coordinates; (3) investigate the symmetries (*x*, and *y*-axes and about pole) of a polar curve using a graphing tool; (4) investigate the geometrical properties (loci, reflection, eccentricity, directrix) of a conic section using a dynamic geometry tool;

	Topic / Sub-topic and Content	Learning Experiences and Applications
	$\frac{(y-k)^2}{b^2}-\frac{(x-h)^2}{a^2}=1$ • Conic sections in polar form given by $r=\frac{ep}{1\pm e\cos\theta}$ or $r=\frac{ep}{1\pm e\sin\theta}$, where $e>0$ is the eccentricity and $\lvert p \rvert$ is the distance between the focus (pole) and the directrix	(5) explain the proofs of the geometrical properties of conic sections; and (6) model and solve problems related to parabolic and hyperbolic mirrors, and orbits of planets.
1.4	Applications of definite integrals • Use of formula $A=\frac{1}{2}\int_{\alpha}^{\beta} r^2 \, \mathrm{d}\theta$ for the area of a sector • Arc length of curves defined in cartesian, parametric or polar form • Volume of revolution about the x- or y-axis for curves defined in cartesian or parametric form using discs or shells as appropriate • Surface area of revolution about the x- or y-axis for curves defined in cartesian or parametric form	Examples of what students would do as part of their learning: (1) read and explain the derivation of the cylindrical shell method; (2) identify when the cylindrical method is more appropriate than the disc method; (3) compare different ways of finding areas of circles and ellipses using both the polar and Cartesian forms; (4) solve problems related to the flight path of a projectile, work done or turning force; and (5) model and solve problems related to the concept of mean value such as finding the centre of mass, moment of inertia of a solid object.
1.5	Differential equations • Analytical solution of first order and second order linear differential equations of the form: (i) $\frac{\mathrm{d}y}{\mathrm{d}x}=\mathrm{f}(x)\,\mathrm{g}(y)$ (ii) $\frac{\mathrm{d}y}{\mathrm{d}x}+\mathrm{p}(x)y=\mathrm{q}(x)$, using an integrating factor (iii) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+a\frac{\mathrm{d}y}{\mathrm{d}x}+by=0$ (iv) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+a\frac{\mathrm{d}y}{\mathrm{d}x}+by=\mathrm{f}(x)$, where $\mathrm{f}(x)$ is a polynomial or pe^{kx} or $p\cos(kx)+q\sin(kx)$ including those that can be reduced to the above by means of a given substitution • Relationship between the solution of a non-homogenous equation and the associated homogenous equation • Family of solution curves	Examples of what students would do as part of their learning: (1) explore and discuss the characteristics of a family of solution curves using a graphing tool e.g. the locus of stationary points of $\frac{\mathrm{d}y}{\mathrm{d}x}+\frac{1}{x}y=x$ lies on the curve $y=x^2$; (2) model and solve problems related to the spread of diseases or population growth, with competition and harvesting; and (3) model and solve problems related to the motion of particles that involves resistance, free or driven oscillation and damping.

	Topic / Sub-topic and Content	Learning Experiences and Applications
	• Exponential growth model • Logistic growth model with harvesting	
2	**Discrete Mathematics, Matrices and Numerical Methods**	
2.1	Recurrence relations • Sequence generated by a simple recurrence relation, including the use of graphing calculator to generate the sequence defined by the recurrence relation • Behaviour of a sequence, such as the limiting behaviour of a sequence • Solution of (i) First order linear (homogeneous and non-homogeneous) recurrence relations with constant coefficients of the form $u_n = au_{n-1} + b, a, b \in \mathbb{R}, a \neq 0$ (ii) Second order linear homogeneous recurrence relations with constant coefficients • Modelling with recurrence relations of the forms above	Examples of what students would do as part of their learning: (1) relate the concepts of arithmetic and geometric progressions to the concepts of recurrence relations; (2) investigate the behaviour of a sequence defined by a recurrence relation of the form $x_{n+1} = \mathrm{f}(x_n)$ and $u_{n+1} = au_n + b, b \in \mathbb{R}, a \neq 0$ using a spreadsheet or an equivalent tool; (3) formulate the Fibonacci sequence as a second order recurrence relation; (4) relate the solutions of recurrence relations to those of corresponding differential equations; (5) model and solve problems involving population growth, investment or loan on a discrete time scale; and (6) compare discrete and continuous models and discuss the advantages or disadvantages of both.
2.2	Matrices and linear spaces • Use of matrices to represent a set of linear equations • Operations on 3×3 matrices • Determinant of a square matrix and inverse of a non-singular matrix (2×2 and 3×3 matrices only) • Use of matrices to solve a set of linear equations (including row reduction and echelon forms, and geometrical interpretation of the solution) • Linear spaces and subspaces, and the axioms (restricted to spaces of finite dimension over the field of real numbers only) • Linear independence • Basis and dimension (in simple cases), including use of terms such as 'column space', 'row space', 'range space' and 'null space' • Rank of a square matrix and relation between rank, dimension of null space and order of the	Examples of what students would do as part of their learning: (1) relate the concept of the solution space of a system linear equations to the intersection of planes; (2) relate the concept of rank, dimension of the null space and the order of the 3×3 matrix to the relationship and intersection of planes; (3) apply the concept of eigenvalues and eigenvectors to find higher powers of a matrix used to model a recurrence relation; and (4) read about the use of matrices in the computation of the Page Rank in the Google search engine.

	Topic / Sub-topic and Content	Learning Experiences and Applications
	matrix • Linear transformations and matrices from $\mathbb{R}^n \to \mathbb{R}^m$ • Eigenvalues and eigenvectors of square matrices (2×2 and 3×3 matrices, restricted to cases where the eigenvalues are real and distinct) • Diagonalisation of a square matrix M by expressing the matrix in the form $\mathbf{QDQ}^{-1}$, where **D** is a diagonal matrix of eigenvalues and **Q** is a matrix whose columns are eigenvectors, and use of this expression such as to find the powers of M	
2.3	Numerical methods • Location of roots of an equation by simple graphical or numerical methods • Approximation of roots of equations using linear interpolation and Newton-Raphson method, including cases where each method fails to converge to the required root • Iterations involving recurrence relations of the form $x_{n+1} = \mathrm{F}(x_n)$, including cases where the method fails to converge • Approximation of integral of a function using the trapezium rule and Simpson's rule • Approximation of solutions of first order differential equations using Euler method (including the use of the improved Euler formula)	Examples of what students would do as part of their learning: (1) suggest ways to find the solution of $x = \cos x$; (2) propose methods of obtaining approximate solutions to equations with no exact solution, before learning numerical methods; (3) implement linear interpolation or Newton-Raphson method using a spreadsheet for the simple case of $x = \cos x$; and (4) implement Euler and improved Euler method for a simple differential equation using a spreadsheet or by writing a program.
3	**Probability and Statistics**	
3.1	Discrete random variables • Use of Poisson distribution Po(μ) and geometric distribution Geo(p) as probability models, including conditions under which each distribution is a suitable model • Mean and variance for Poisson and geometric distributions • Additive property of the Poisson distribution	Examples of what students would do as part of their learning: (1) prove the probability distribution functions for Poisson and geometric distributions and derive their means and variances; (2) verify that the binomial distribution converges to a Poisson distribution as n increases with np being constant using a statistical tool; (3) prove the "memory-less property" of the geometric distribution and explain its implications on real-world situations; (4) model and solve problems related to the number of occurrences of a rare event e.g.

	Topic / Sub-topic and Content	Learning Experiences and Applications
		accident, infection, reliability; and (5) model and solve problems related to the waiting time of an event e.g. game of chance, lifespan in the discrete time scale.
3.2	Continuous random variables • Probability density function of a continuous random variable and its mean and variance (includes 'piecewise' probability density function) • Cumulative distribution function and its relationship with the probability density function • Concepts of median and mode of a continuous random variable • Use of the result $\mathrm{E}(\mathrm{g}(X)) = \int_{-\infty}^{\infty} \mathrm{g}(x)\mathrm{f}(x)\,\mathrm{d}x$ in simple cases, where $\mathrm{f}(x)$ is the probability density function of X and $\mathrm{g}(x)$ is a function of X • Uniform distribution and exponential distribution as probability models • Relationship between Poisson and exponential distributions	Examples of what students would do as part of their learning: (1) prove the "memory-less property" of the exponential distribution and explain its implications on real-world situations; and (2) model and solve problems related to the waiting time of an event e.g. game of chance, lifespan in the continuous time scale.
3.3	Hypothesis testing and Confidence intervals • Formulation of hypotheses and testing for a population mean using a small sample drawn from a normal population of unknown variance using a *t*-test • Formulation of hypotheses for the difference of population means, and apply, as appropriate: - a 2-sample *t*-test - a paired sample *t*-test - a test using a normal distribution • Contingency tables and χ^2 -tests of: - goodness of fit - independence (excluding Yates' correction for continuity) • Connection between confidence interval and hypothesis test • Confidence interval for the population mean based on: - a random sample from a normal population of known variance - a small random sample drawn from a normal population of unknown variance	Examples of what students would do as part of their learning: (1) suggest ways to determine if a sample comes from a theoretical distribution based on the data given before teaching the goodness-of-fit test; (2) suggest ways to determine if two categories are independent based on the data given before teaching the test of independence; (3) apply the tests to problems in science or social experiments e.g. effect of a new drug, intervention program or impact of an advertisement; (4) suggest ways to bound the population mean based on data from a sample method before introducing the concept of confidence intervals; (5) reason intuitively that the length of the confidence intervals should increase or decrease with the sample size and variance before learning the formula; and (6) simulate confidence intervals of the mean

	Topic / Sub-topic and Content	Learning Experiences and Applications
	- a large random sample from any population • Confidence interval for population proportion (including concept of sample proportion) from a large random sample • Interpretation of confidence intervals and the results of a hypothesis test in the context of the problem Exclude the use of the term ‘Type I error’, concept of Type II error and power of a test.	from different samples to verify the proportion of intervals that contain the mean.
3.4	Non-parametric tests • Sign test • Wilcoxon matched-pair signed rank test • Advantages and disadvantages of non-parametric tests	Examples of what students would do as part of their learning: (1) discuss situations where a non-parametric test is more suitable than a parametric one; and (2) compare the results from a non-parametric and a parametric test.

3. PEDAGOGY

Principles of Teaching and Learning

The following principles guide the teaching and learning of mathematics.

- Principle 1: Teaching is for learning; learning is for understanding; understanding is for reasoning and applying and, ultimately problem solving.

- Principle 2: Teaching should build on the pre-requisite knowledge for the topics; take cognisance of students' interests and experiences; and engage them in active and reflective learning.

- Principle 3: Teaching should connect learning to the real world, harness technology and emphasise 21st century competencies.

These principles capture the importance of deep and purposeful learning, student-centric pedagogies and self-directed learning, relevance to the real world, learning with technology and future orientation towards learning.

Learning Experiences

Learning mathematics is more than just learning concepts and skills. Equally important are the cognitive and metacognitive process skills. These processes are learned through carefully constructed learning experiences. The learning experiences stated in Section 2 of the syllabus link the learning of content to the development of mathematical processes. They are examples of what students would do as part of their learning. These learning experiences create opportunities for students to:

a) Engage in mathematical discourse where students actively discuss and construct mathematical arguments and proofs (e.g. critiquing each other's proof and argument) and reason and communicate their understanding using precise mathematical language;

b) Study a wide range of real-world problems (e.g. using logistic model to study the spread of a disease) afforded by the concepts and models in the syllabus, with deeper discussion of the limitations of the model and be engaged in mathematical modelling tasks, individually or in groups; and

c) Read and discuss mathematics articles that deepen their understanding of concepts and appreciation of the relevance of mathematics to the real world (e.g. article on exploring how matrices, eigenvalues and eigenvectors play a part in ranking Internet search results, how differential equations can be used in determining the authenticity of paintings).

The learning experiences also contribute to the development of 21CC. For example, to encourage students to be inquisitive, the learning experiences include opportunities where students discover mathematical results on their own. To support the development of collaborative and communication skills, students are given opportunities to work together on a problem and present their ideas using appropriate mathematical language and methods. To develop habits of self-directed learning, students are given opportunities to set learning goals and work towards them purposefully.

Teaching and Learning Approaches

To better cater to the learning needs of JC students and to equip them with 21CC, students would experience a blend of pedagogies. Pedagogies that are constructivist in nature complement direct instruction. A constructivist classroom features greater student participation, collaboration and discussion, and greater dialogue between teachers and peers. Students take on a more active role in learning, and construct new understandings and knowledge. The teacher's role is to facilitate the learning process (e.g. through more in-depth dialogue and questioning) and guide students to build on their prior knowledge, and provide them with opportunities for more ownership and active engagement during learning.

Below are examples of possible strategies that support the constructivist approach to learning:

- Activity based learning e.g. individual or group work, problem solving
- Teacher-directed inquiry e.g. demonstration, posing questions
- Flipped classroom e.g. independent study, followed by class discussion
- Seminar e.g. mathematical discussion and discourse
- Case studies e.g. reading articles, analysing real data
- Project e.g. mathematical modelling, statistical investigation
- Lab work e.g. simulation, investigation using software and application

4. ASSESSMENT

Role of Assessment

The role of assessment is to improve teaching and learning. For students, assessment provides them with information about how well they have learned and how they can improve. For teachers, assessment provides them with information about their students' learning and how they can adjust their instruction. Assessment is therefore an integral part of the interactive process of teaching and learning.

Assessment in mathematics should focus on students':

- understanding of mathematics concepts (going beyond simple recall of facts);
- ability to draw connections and integrate ideas across topics;
- capacity for logical thought, particularly, the ability to reason, communicate, and interpret; and
- ability to formulate, represent and solve problems within mathematics and other contexts.

The purpose of assessments can be broadly classified as summative, formative, and diagnostic.

- Summative assessments, such as tests and examinations, measure what students have learned. Students will receive a score or a grade.
- Formative and diagnostic assessments are used to support learning and to provide timely feedback for students on their learning, and to teachers on their teaching.

Classroom Assessments

Assessments in the mathematics classroom are primarily formative and diagnostic in purpose. Classroom assessments include the questions teachers asked during lessons, the homework assigned to students, and class tests given at different times of the academic year. For these assessments to be formative, feedback to students is important. Students should use the feedback from these assessments to understand where they are in their learning and how to improve their learning.

GCE A-Level National Examination

Students will take the national examination in their final year. The national examination is a summative assessment that measures the level of attainment of the outcomes stated in the syllabuses.

The national examination code for the paper is **9649**. The examination syllabus can be found in the SEAB website. Important information about the examination is reproduced here.

Assessment Objectives (AO)

The assessment will test students' abilities to:

AO1 Understand and apply a wide range of mathematical concepts and skills in a variety of problems, including those that may be set in unfamiliar contexts, or require integration of concepts and skills from more than one topic.

AO2 Formulate real-world problems mathematically, solve the mathematical problems, interpret and evaluate the mathematical solutions in the context of the problems.

AO3 Reason and communicate mathematically through forming conjectures, making deductions and constructing rigorous mathematical arguments and proofs.

The examinations will be based on the topic/sub-topic and content list on page 9 – 14. Knowledge of the content of *H2 Mathematics* is assumed.

Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics.

While problems may be set based in contexts, no assumptions will be made about the knowledge of the contexts. All information will be self-contained within the problem.

Scheme of Examination Papers

There will be two 3-hour papers, each carrying 50% of the total mark, and each marked out of 100, as follows:

PAPER 1 (3 hours)
A paper consisting of 10 to 12 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

There will be at least two questions on application of Mathematics in real-world contexts, including those from sciences and engineering. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Students will be expected to answer **all** questions.

PAPER 2 (3 hours)
A paper consisting of 2 sections, Sections A and B.

Section A (Pure Mathematics – 50 marks) will consist of 5 to 6 questions of different lengths and marks based on the Pure Mathematics section (i.e. Algebra & Calculus, and Discrete Mathematics, Matrices & Numerical Methods) of the syllabus.
Section B (Probability and Statistics – 50 marks) will consist of 5 to 6 questions of different lengths and marks based on the Probability and Statistics section of the syllabus.

There will be at least two questions in Section B on application of Mathematics in real-world contexts, including those from sciences and engineering. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Students will be expected to answer **all** questions.

Use of a graphing calculator (GC)

The use of an approved GC *without* computer algebra system will be expected. The examination papers will be set with the assumption that students will have access to GC. As a general rule, unsupported answers obtained from GC are allowed unless the question states otherwise. Where unsupported answers from GC are not allowed, students are required to present the mathematical steps using mathematical notations and not calculator commands. For questions where graphs are used to find a solution, students should sketch these graphs as part of their answers. Incorrect answers without working will receive no marks. However, if there is written evidence of using GC correctly, method marks may be awarded.

Students should be aware that there are limitations inherent in GC. For example, answers obtained by tracing along a graph to find roots of an equation may not produce the required accuracy.

List of formulae and statistical tables

Students will be provided in the examination with a list of formulae and statistical tables.

Mathematical notation

A list of mathematical notation is available at the SEAB website.

5. USEFUL REFERENCE BOOKS

Pure Mathematics

- Akopyan, A. V. & Zaslavsky, A. A. (2007). Geometry of conics (Mathematical world). American Mathematical Society.
- Anton, H. (2013). Elementary linear algebra. (11^{th} Ed.). Wiley.
- Blank, B. E. & Krantz, S. G. (2011). Calculus: Single variable. (4^{th} Ed.). Wiley.
- Kendig, K. (2005). Conics. The Mathematical Association of America.
- Lay, D. C., Lay, S. R. & McDonald J. J. (2015). Linear algebra and its applications. (5^{th} Ed.). Pearson.
- Rosen, K. (2011). Discrete mathematics and its applications. (7^{th} Ed.). McGraw-Hill Education.
- Stewart, J. (2015). Calculus: Early transcendentals. (8^{th} Ed.). Cengage Learning.
- Zill, D. G. (2013). A first course in differential equations with modelling applications. (10^{th} Ed.). Brooks Cole.

Probability and Statistics

- Agresti, A. & Franklin, C. A. (2012). Statistics: The art and science of learning from data. (3^{rd} Ed.). Pearson.
- Crawshaw, J. & Chambers, J. (2001). A concise course in advanced level statistics. (4^{th} Ed.). Nelson Thornes Ltd.
- Dobbs, S. & Miller, J. (2003). Statistics 1, 2. Cambridge University Press.
- Freedman, D., Pisani, R. & Purves, R. (2007). Statistics. (4^{th} Ed.). W. W. Norton & Company.
- McClave, J. T. & Sincich, T. T. (2012). Statistics. (12^{th} Ed.). Pearson.
- Walpole, R. E., Myers, R. H., Myers, S. L. & Ye, K. (2012). Probability & statistics for engineers & scientists. (9^{th} Ed.). Pearson.

*The list is by no means exhaustive as they provide some samples that students can refer to. There are other reference books that students can use as well.