MATHEMATICS SYLLABUS

Pre-University

H1 Mathematics

Implementation starting with
2016 Pre-University One Cohort

Ministry of Education
SINGAPORE

CONTENTS

1. INTRODUCTION

Importance of Mathematics

Mathematics contributes to the developments and understanding in many disciplines. It is used extensively to model the real world, create new products and services and support data-driven decisions. A good foundation in mathematics and a keen appreciation of its potential give one a competitive edge over others.

Discipline of Mathematics

Mathematics is a study about quantities, space, patterns, relationships, chance and abstractions. Mathematical knowledge is established through rigorous proofs, derived from axioms and definitions through logical argument and reasoning. Mathematical statements or claims should be challenged and remain as conjectures until they are proven to be true.

Mathematics can be seen as a language. It is used to express, communicate and share ideas, within the scientific communities as well as with the general public. It has its own set of notations, symbols, and terminologies. It is a language that strives to be precise and concise.

The applications of mathematics transcend its own boundary, into the daily life, the real world and other disciplines. It is more than just computations. Mathematics is a powerful tool to model real world phenomena. But it has its limitations, as often mathematical models cannot capture all the complexities of real world.

Learning of Mathematics

The learning of mathematics should honour the nature of the discipline and its practices. Students should therefore learn to justify their solutions, give reasons to support their conclusions and prove mathematical statements. They should also learn to communicate mathematically, construct and discuss mathematical statements, and use the language of mathematics to develop and follow a logical chain of reasoning. In applying mathematics to solve real world problems, they should learn to formulate models, be aware of the limitations of these models and exercise care in the interpretation of mathematics solutions. Such learning experiences will provide students a glimpse of what being a mathematician is like and what mathematics is about.

Mathematics at the A-Level

In Singapore, mathematics education at the A-level plays an important role in laying the foundation for building a pool of highly skilled and analytical workforce, especially in STEM-related areas. From the period of rapid industrialisation in the 80's to the current day of knowledge intensive industries, it continues to be highly valued by stakeholders and students preparing for tertiary education. Although mathematics is an optional subject at the A-level, it is offered by nearly all students.

The purpose of learning mathematics at the A-level is two-fold. Firstly, it provides students, regardless of the intended course of study at the university, with a useful set of tools and problem solving skills to support their tertiary study. Secondly, learning mathematics exposes students to a way of thinking that complements the ways of thinking developed through other disciplines. This contributes to the development of a well-rounded individual who is able to think deeply, broadly and differently about problems and issues.

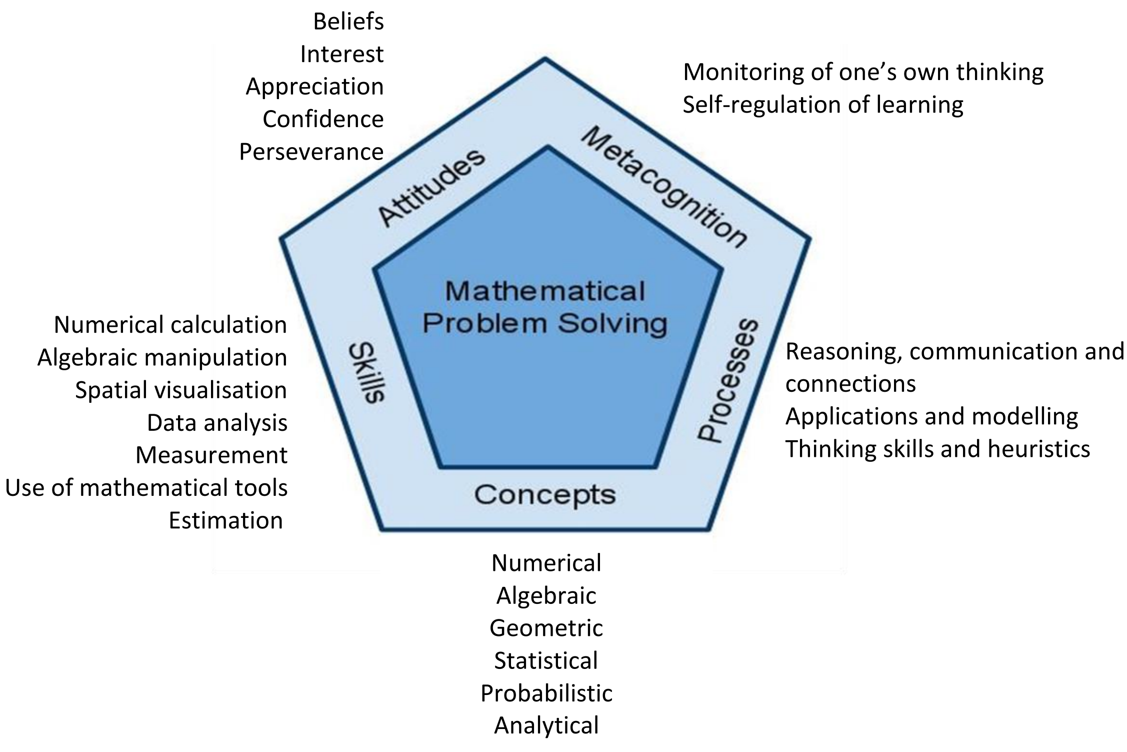
A suite of syllabuses is available to students at the A-level. The syllabuses are:

- H1 Mathematics;
- H2 Mathematics;
- H2 Further Mathematics; and
- H3 Mathematics.

The suite of syllabuses is designed for different profiles of students, to provide them with options to learn mathematics at different levels, and to varying breadth, depth or specialisation so as to support their progression to their desired choice of university courses.

Mathematics Framework

The Mathematics Framework sets the direction for curriculum and provides guidance in the teaching, learning, and assessment of mathematics. The central focus is mathematical problem solving, that is, using mathematics to solve problems. The curriculum stresses *conceptual understanding*, *skills proficiency* and *mathematical processes*, and gives due emphasis to *attitudes* and *metacognition*. These five components are inter-related.



- *Concepts*

At the A-level, students continue to study concepts and skills in the major strands of mathematics, which provide the building blocks for the learning of advanced mathematics, with varying breadth and depth depending on the syllabuses. These major strands include Algebra, Calculus, Vectors, and Probability and Statistics, which are rich in applications within mathematics and in other disciplines and the real world. These content categories are connected and interdependent.

- *Skills*

Mathematical skills refer to *numerical calculation*, *algebraic manipulation*, *spatial visualisation*, *data analysis*, *measurement*, *use of mathematical tools*, and *estimation*. The skills are specific to mathematics and are important in the learning and application of mathematics. In today's classroom, these skills also include the abilities to use spreadsheets and other software to learn and do mathematics.

- *Processes*

Mathematical processes refer to the skills involved in acquiring and applying mathematical knowledge. These include *reasoning, communication and connections*, *applications and modelling*, and *thinking skills and heuristics* that are important in mathematics.

Reasoning, communication and connections

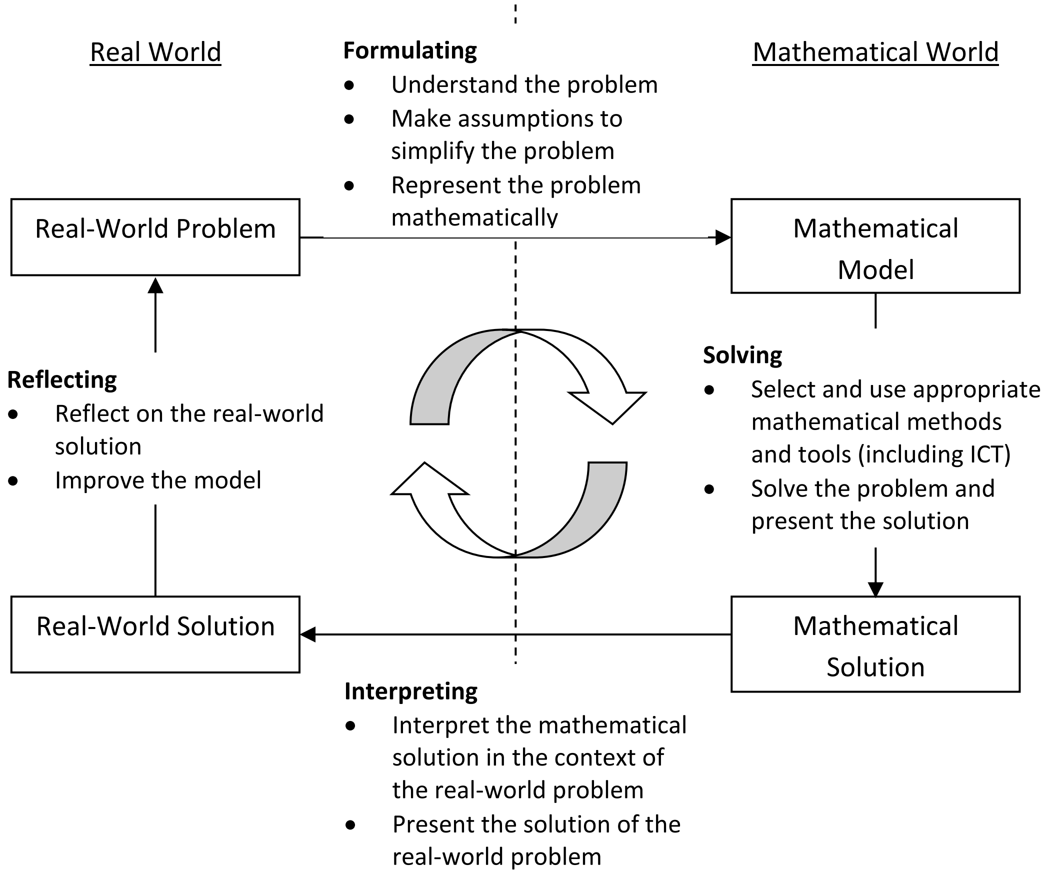
- Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments.
- Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically.
- Connections refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and the real world.

Applications and modelling

Exposing students to applications and modelling enhances their understanding and appreciation of mathematics. Mathematical modelling is the process of formulating and improving a mathematical model[1] to represent and solve real-world problems. Through mathematical modelling, students learn to deal with complexity and ambiguity by simplifying and making reasonable assumptions, select and apply appropriate mathematical concepts and skills that are relevant to the problems, and interpret and evaluate the solutions in the context of the real-world problem.

[1] A mathematical model is a mathematical representation or idealisation of a real-world situation. It can be as complicated as a system of equations or as simple as a geometrical figure. As the word "model" suggests, it shares characteristics of the real-world situation that it seeks to represent.

Mathematical Modelling Process



Thinking skills and heuristics

Thinking skills refers to the ability to classify, compare, analyse, identify patterns and relationships, generalise, deduce and visualise. Heuristics are general strategies that students can use to solve non-routine problems. These include using a representation (e.g. drawing a diagram, tabulating), making a guess (e.g. trial and error/ guess and check, making a supposition), walking through the process (e.g. working backwards) and changing the problem (e.g. simplifying the problem, considering special cases).

- *Metacognition*

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning.

- *Attitudes*

Attitudes refer to the affective aspects of mathematics learning such as:

- beliefs about mathematics and its usefulness;
- interest and enjoyment in learning mathematics;
- appreciation of the beauty and power of mathematics;

- confidence in using mathematics; and
- perseverance in solving a problem.

In the A-level mathematics curriculum, there is an emphasis on the development of mathematical processes, in particular, reasoning, communications and modelling.

Mathematics and 21st Century Competencies (21CC)

Learning mathematics (undergirded by the Mathematics Framework) supports the development of 21CC and the Desired Outcomes of Education. Students will have opportunities to experience mathematical investigation, reasoning, modelling and discourse, working individually as well as in groups, and using ICT tools where appropriate in the course of learning and doing mathematics. Through these experiences, students learn to think critically and inventively about the problems and their solutions, communicate and collaborate effectively with their peers in the course of learning, use technological tools and manage information[2]. The choice of contexts for the problems in the various syllabuses can help raise students' awareness of local and global issues around them. For example, problems set around population issues and health issues can help students understand the challenges faced by Singapore and those around the world[3]. Assessment will also play a part in encouraging students to pay attention to the 21CC. Classroom and national assessment would require students to think critically and inventively and communicate and explain their reasons effectively when they solve problems; and not just recalling formulae and procedures and performing computations.

[2] These opportunities, e.g. thinking critically and inventively, collaborating effectively with their peers are related to the Desired Outcomes of Education: A confident person, a self-directed learner, and an active contributor.
[3] These are related to the Desired Outcomes of Education: A concerned citizen.

2. CONTENT: H1 MATHEMATICS (FROM 2016)

Preamble

The applications of mathematics extend beyond the sciences and engineering domains. A basic understanding of mathematics and statistics, and the ability to think mathematically and statistically are essential for an educated and informed citizenry and in other fields of study.

H1 Mathematics provides students with a foundation in mathematics and statistics that will support their business or social sciences studies at the university. It is particularly appropriate for students without an *Additional Mathematics* background because it offers an opportunity for them to learn important mathematical concepts and skills in algebra and calculus that were taught in *Additional Mathematics*. Students will also learn basic statistical methods that are necessary for studies in business and social sciences.

Syllabus Aims

The aims of *H1 Mathematics* are to enable students to:

(a) acquire mathematical concepts and skills to support their tertiary studies in business and the social sciences;

(b) develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving;

(c) connect ideas within mathematics and apply mathematics in the context of business and social sciences; and

(d) experience and appreciate the value of mathematics in life and other disciplines.

Content Description

There are 3 content strands in *H1 Mathematics*, namely, *Functions and Graphs*, *Calculus*, and *Probability and Statistics*.

a) <u>Functions and Graphs</u> provides the foundation for algebraic and quantitative reasoning and includes useful topics such as exponential and logarithmic functions, graphing techniques and tools (e.g. graphing calculators), techniques for solving equations, inequalities and system of equations.

b) <u>Calculus</u> provides useful tools for analysing and modelling change and behaviour, and includes basic differentiation and integration concepts, techniques and applications such as finding optimal value and area under a curve.

c) <u>Probability and Statistics</u> provides the foundation for modelling chance phenomena and making inferences with data and includes an introduction to counting techniques, computation of probability, binomial and normal distributions, sampling and hypothesis testing as well as correlation and regression.

There are many connections that can be made between the topics within each strand and across strands, even though the syllabus content are organised in strands. These connections will be emphasised so as to enable students to integrate the concepts and skills in a coherent manner to solve problems.

Knowledge of the content of *O-Level Mathematics* syllabus is assumed in this syllabus.

Applications and Contexts

As *H1 Mathematics* is designed for students who intend to pursue further studies in business and social sciences courses, students will be exposed to the applications of mathematics in business and social sciences, so that they can appreciate the value and utility of mathematics in these likely courses of study.

The list below illustrates the kinds of contexts that the mathematics learnt in the syllabus may be applied. It is by no means exhaustive.

Applications and contexts	**Some possible topics involved**
Optimisation problems (e.g. maximising profits, minimising costs)	Inequalities; System of linear equations; Calculus
Population growth, radioactive decay	Exponential and logarithmic functions
Financial Mathematics (e.g. profit and cost analysis, demand and supply, banking, insurance)	Equations and inequalities; Probability; Sampling distributions; Correlation and regression
Games of chance, elections	Probability
Standardised testing	Normal distribution; Probability
Market research (e.g. consumer preferences, product claims)	Sampling distributions; Hypothesis testing; Correlation and regression
Clinical research (e.g. correlation studies)	Sampling distributions; Hypothesis testing; Correlation and regression

While students will be exposed to applications and contexts beyond mathematics, they are not expected to learn them in depth. Students should be able to use given information to formulate and solve the problems, applying the relevant concepts and skills and interpret the solution in the context of the problem.

	Topic / Sub-topic and Content	Learning Experiences and Applications
1	**Functions and Graphs**	
1.1	Exponential and logarithmic functions and Graphing techniques Include: • concept of function as a rule or relationship where for every input there is only one output • use of notations such as $f(x) = x^2 + 5$ • functions e^x and $\ln x$ and their graphs • exponential growth and decay • logarithmic growth • equivalence of $y = e^x$ and $x = \ln y$ • laws of logarithms • use of a graphing calculator to graph a given function • characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes (horizontal and vertical) Exclude: • use of the terms domain and range • the use of notation $f : x \mapsto x^2 + 5$	Examples of what students would do as part of their learning: (1) discuss examples and non-examples of functions in daily life; (2) examine examples of functions presented in different forms, as an algebraic expression, as a graph or as a table; (3) relate the laws of indices to the laws of logarithms and explain how one can be obtained from the other; (4) explore horizontal and vertical asymptotes using graphing tools and appreciate the limitation of such tools in handling asymptotes; (5) suggest an algebraic expression for a function based on the characteristics of a given graph; (6) model and solve problems involving investment and loans, future and present values; and (7) model and solve problems involving profit, cost and demand functions and their graphs.
1.2	Equations and inequalities Include: • conditions for a quadratic equation to have (i) two real roots, (ii) two equal roots, and (iii) no real roots • conditions for $ax^2 + bx + c$ to be always positive (or always negative) • solving simultaneous equations, one linear and one quadratic, by substitution • solving quadratic equations and inequalities in one unknown analytically • solving inequalities by graphical methods • formulating an equation or a system of linear equations from a problem situation • finding the approximate solution of an equation or a system of linear equations using a graphing calculator	Examples of what students would do as part of their learning: (1) explain how the nature of roots of a quadratic equation is related to the sign of the discriminant; (2) explore and explain how the sign of the leading coefficient determine whether the graph has a maximum or a minimum point; (3) discuss the solution(s), or lack of it, of a system of linear equations in the context of the problem; (4) discuss the limitations of using a graphing calculator to obtain the answers to a problem; (5) model and solve problems involving profit functions that are quadratic in nature; and (6) model and solve problems involving break-even analysis, revenue and cost functions.

	Topic / Sub-topic and Content	Learning Experiences and Applications
2	**Calculus**	
2.1	Differentiation Include: • derivative of $\mathrm{f}(x)$ as the gradient of the tangent to the graph of $y=\mathrm{f}(x)$ at a point • use of standard notations $\mathrm{f}'(x)$ and $\frac{\mathrm{d}y}{\mathrm{d}x}$ • derivatives of x^n for any rational n, e^x, $\ln x$, together with constant multiples, sums and differences • use of chain rule • graphical interpretation of $\mathrm{f}'(x)>0$, $\mathrm{f}'(x)=0$ and $\mathrm{f}'(x)<0$ • use of the first derivative test to determine the nature of the stationary points (local maximum and minimum points and points of inflexion) in simple cases • locating maximum and minimum points using a graphing calculator • finding the approximate value of a derivative at a given point using a graphing calculator • finding equations of tangents to curves • local maxima and minima problems • connected rates of change problems Exclude: • differentiation from first principles • derivatives of products and quotients of functions • use of $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$ • differentiation of functions defined implicitly or parametrically • finding non-stationary points of inflexion • relating the graph of $y=\mathrm{f}'(x)$ to the graph of $y=\mathrm{f}(x)$	Examples of what students would do as part of their learning: (1) suggest ways to define the rate of change at a given point; (2) verify that the slope of a line segment joining (x, f(x)) and (x+h, f(x+h)) converges to $\mathrm{f}'(x)$ as $h \to 0$ using a spreadsheet or graphing tool; (3) describe exponential or logarithmic growth in terms of the rate of change; (4) model and solve problems involving marginal profit/cost; (5) model and solve problems involving optimisation e.g. finding optimal selling price; and (6) model and solve problems involving population growth, investment and loans.
2.2	Integration Include: • integration as the reverse of differentiation • integration of x^n for any rational n, and e^x, together with constant multiples, sums and differences	Examples of what students would do as part of their learning: (1) explain the need for an arbitrary constant in indefinite integrals; (2) deduce indefinite integrals by trial and error and generalise them; (3) verify that the area under a straight line

	Topic / Sub-topic and Content	Learning Experiences and Applications
	• integration of $(ax+b)^n$ for any rational n, and $e^{(ax+b)}$ • definite integral as the area under a curve • evaluation of definite integrals • finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves • finding the approximate value of a definite integral using a graphing calculator Exclude: • definite integral as a limit of sum • approximation of area under a curve using the trapezium rule • area below the x-axis	graph is given by the definite integral over the same interval; (4) model and solve problems involving net changes or excesses over time; and (5) model and solve problems involving total revenues and earnings over time.
3	**Probability and Statistics**	
3.1	Probability Include: • addition and multiplication principles for counting • concepts of permutation (${}^nP_r$) and combination (${}^nC_r$) • simple problems on arrangement and selection including cases involving repetition and restriction • addition and multiplication of probabilities • mutually exclusive events and independent events • use of tables of outcomes, Venn diagrams, tree diagrams, and permutations or combinations techniques to calculate probabilities • calculation of conditional probabilities in simple cases • use of: $P(A')=1-P(A)$ $P(A\cup B)=P(A)+P(B)-P(A\cap B)$ $P(A\mid B)=\dfrac{P(A\cap B)}{P(B)}$ Exclude: • arrangements of objects in a circle • arrangements of identical objects (use of formula $\dfrac{n!}{n_1!n_2!\cdots n_m!}$)	Examples of what students would do as part of their learning: (1) discuss and generate examples of problems where the addition and multiplication principles are applied; (2) explain the relationship between ${}^nP_r$ and ${}^nC_r$; (3) discuss real-life examples where probability is required; (4) discuss the role of conditional probability in the Monty Hall problem, prosecutors' fallacy or Sally Clark case; (5) compare between mutually exclusive events and independent events; (6) verify that the relative frequency converges to the theoretical probability by simulating the tossing of a fair coin using a random number generator; and (7) model and solve problems involving games of chance, human behaviours, risks or propensity.

	Topic / Sub-topic and Content	Learning Experiences and Applications
3.2	Binomial distribution Include: • knowledge of the binomial expansion of $(a+b)^n$ for positive integer n • binomial random variable as an example of a discrete random variable • concept of binomial distribution $\mathrm{B}(n,p)$ and use of $\mathrm{B}(n,p)$ as a probability model, including conditions under which the binomial distribution is a suitable model • use of mean and variance of a binomial distribution (without proof)	Examples of what students would do as part of their learning: (1) give examples of real-world situations that involve discrete random variables; (2) give examples of real-world situations that can be modelled using a binomial distribution; (3) explain why a given real-world situation cannot be modelled using a binomial distribution; (4) relate the expansion of $(p+q)^n$, where $p+q=1$ and $n \in \mathbb{Z}^+$ to the probability distribution function for a binomial random variable; (5) compute the mean and variance for a simple binomial distribution using a probability distribution table; and (6) explore the effects of varying n and p on the shape of the binomial distribution using a graphing tool.
3.3	Normal distribution Include: • concept of a normal distribution as an example of a continuous probability model and its mean and variance; use of $\mathrm{N}(\mu,\sigma^2)$ as a probability model • standard normal distribution • finding the value of $\mathrm{P}(X < x_1)$ or a related probability given the values of x_1, μ, σ • symmetry of the normal curve and its properties • finding a relationship between x_1, μ, σ given the value of $\mathrm{P}(X < x_1)$ or a related probability • solving problems involving the use of $\mathrm{E}(aX+b)$ and $\mathrm{Var}(aX+b)$ • solving problems involving the use of $\mathrm{E}(aX+bY)$ and $\mathrm{Var}(aX+bY)$, where X and Y are independent Exclude normal approximation to binomial distribution.	Examples of what students would do as part of their learning: (1) compare and approximate the normal and binomial distributions for large n with np being constant using a statistical tool; (2) suggest ways to compare two scores from different tests with the test scores following normal distributions; (3) explore the characteristics of a normal curve: shape, centre, spread, and probability as area under the curve using a graphing tool; and (4) model and solve problems involving social phenomena, quality control and standards.

	Topic / Sub-topic and Content	Learning Experiences and Applications
3.4	Sampling Include: • concepts of population, random and non-random samples • concept of the sample mean $\overline{X}$ as a random variable with $\mathrm{E}(\overline{X})=\mu$ and $\mathrm{Var}(\overline{X})=\frac{\sigma^2}{n}$ • distribution of sample means from a normal population • use of the Central Limit Theorem to treat sample means as having normal distribution when the sample size is sufficiently large • calculation of unbiased estimates of the population mean and variance from a sample, including cases where the data are given in summarised form $\sum x$ and $\sum x^2$, or $\sum(x-a)$ and $\sum(x-a)^2$	Examples of what students would do as part of their learning: (1) discuss ways to perform a random selection from a population; (2) discuss the underlying sampling methods in reports e.g. healthcare and exit polls, and evaluate whether it is random or not; (3) simulate sampling from a population and study the distribution of the sample mean as a random variable using a statistical tool; (4) discuss the advantages of summarising the data in coded form and explain how this affects the mean and standard deviation; and (5) simulate large sampling from a population and verify that the distribution of the sample mean has a bell-shaped distribution (CLT).

3.5	Hypothesis testing
	
	Include:
	• concepts of null hypothesis (H_0) and alternative hypotheses (H_1), test statistic, critical region, critical value, level of significance and *p*-value
	• formulation of hypotheses and testing for a population mean based on:
	  - a sample from a normal population of known variance
	  - a large sample from any population
	• 1-tail and 2-tail tests
	• interpretation of the results of a hypothesis test in the context of the problem
	
	Exclude the use of the term 'Type I error', concept of Type II error and testing the difference between two population means.
		Examples of what students would do as part of their learning:
		
		(1) suggest ways to verify and dispute a claim based on data before introducing the concept of hypothesis testing;
		(2) relate the notion of null and alternative hypothesis to the statement "innocent until proven guilty";
		(3) identify real-world situations where hypothesis testing is evident ;
		(4) explain the meaning and relationship between *p*-value and *Z*-value;
		(5) relate the error of hypothesis testing to the notion of "false positive" and "false negative" in health screening; and
		(6) solve problems involving healthcare, product testing, consumer preferences, lifestyle choices and quality control.

3.6	Correlation and Linear regression Include: • use of scatter diagram to determine if there is a plausible linear relationship between the two variables • correlation coefficient as a measure of the fit of a linear model to the scatter diagram	Examples of what students would do as part of their learning: (1) suggest ways to fit a straight line to a set of data that look linearly related with a straight line; (2) suggest ways to measure how good the straight line fits a set of data;

	Topic / Sub-topic and Content	Learning Experiences and Applications
	• finding and interpreting the product moment correlation coefficient (in particular, values close to -1, 0 and 1) • concepts of linear regression and method of least squares to find the equation of the regression line • concepts of interpolation and extrapolation • use of the appropriate regression line to make prediction or estimate a value in practical situations, including explaining how well the situation is modelled by the linear regression model Exclude: • derivation of formulae • relationship $r^2 = b_1 b_2$, where b_1 and b_2 are regression coefficients • hypothesis tests • use of a square, reciprocal or logarithmic transformation to achieve linearity	(3) draw the least squares line for the set of 9 points $(0,0)$, $(0,1)$, $(0,-1)$, $(1,0)$, $(-1,0)$, $(1,1)$, $(1,-1)$, $(-1,1)$, $(-1,-1)$, without computing the line; (4) discuss the difference between correlation and causation; (5) contrast interpolation and extrapolation and discuss the risk of extrapolation though real-life examples; and (6) investigate the relationship between the two regression lines (y-on-x and x-on-y).

3. PEDAGOGY

Principles of Teaching and Learning

The following principles guide the teaching and learning of mathematics.

- Principle 1: Teaching is for learning; learning is for understanding; understanding is for reasoning and applying and, ultimately problem solving.

- Principle 2: Teaching should build on the pre-requisite knowledge for the topics; take cognisance of students' interests and experiences; and engage them in active and reflective learning.

- Principle 3: Teaching should connect learning to the real world, harness technology and emphasise 21st century competencies.

These principles capture the importance of deep and purposeful learning, student-centric pedagogies and self-directed learning, relevance to the real world, learning with technology and future orientation towards learning.

Learning Experiences

Learning mathematics is more than just learning concepts and skills. Equally important are the cognitive and metacognitive process skills. These processes are learned through carefully constructed learning experiences. The learning experiences stated in Section 2 of the syllabus link the learning of content to the development of mathematical processes. They are examples of what students would do as part of their learning. These learning experiences create opportunities for students to:

a) Engage in mathematical discussion where students actively reason and communicate their understanding to their peers and solve problems collaboratively;

b) Construct mathematical concepts (e.g. to develop their own measure of linear relationship before being taught the formal concept) and form their own understanding of the concepts;

c) Model and apply mathematics to a range of real-world problems (e.g. using exponential growth model to model population growth) afforded by the concepts and models in the syllabus;

d) Make connections between ideas in different topics and between the abstract mathematics and the real-world applications and examples; and

e) Use ICT tools to investigate, form conjecture and explore mathematical concepts (e.g. properties of graphs and their relationship with the algebraic expressions that describe the graph, the Central Limit Theorem).

The learning experiences also contribute to the development of 21CC. For example, to encourage students to be inquisitive, the learning experiences include opportunities where students discover mathematical results on their own. To support the development of collaborative and communication skills, students are given opportunities to work together on a problem and present their ideas using appropriate mathematical language and methods. To develop habits of self-directed learning, students are given opportunities to set learning goals and work towards them purposefully.

Teaching and Learning Approaches

To better cater to the learning needs of JC students and to equip them with 21CC, students would experience a blend of pedagogies. Pedagogies that are constructivist in nature complement direct instruction. A constructivist classroom features greater student participation, collaboration and discussion, and greater dialogue between teachers and peers. Students take on a more active role in learning, and construct new understandings and knowledge. The teacher's role is to facilitate the learning process (e.g. through more in-depth dialogue and questioning) and guide students to build on their prior knowledge, and provide them with opportunities for more ownership and active engagement during learning.

Below are examples of possible strategies that support the constructivist approach to learning:

- Activity based learning e.g. individual or group work, problem solving
- Teacher-directed inquiry e.g. demonstration, posing questions
- Flipped classroom e.g. independent study, followed by class discussion
- Seminar e.g. mathematical discussion and discourse
- Case studies e.g. reading articles, analysing real data
- Project e.g. mathematical modelling, statistical investigation
- Lab work e.g. simulation, investigation using software and application

4. ASSESSMENT

Role of Assessment

The role of assessment is to improve teaching and learning. For students, assessment provides them with information about how well they have learned and how they can improve. For teachers, assessment provides them with information about their students' learning and how they can adjust their instruction. Assessment is therefore an integral part of the interactive process of teaching and learning.

Assessment in mathematics should focus on students':

- understanding of mathematics concepts (going beyond simple recall of facts);
- ability to draw connections and integrate ideas across topics;
- capacity for logical thought, particularly, the ability to reason, communicate, and interpret; and
- ability to formulate, represent and solve problems within mathematics and other contexts.

The purpose of assessments can be broadly classified as summative, formative, and diagnostic.

- Summative assessments, such as tests and examinations, measure what students have learned. Students will receive a score or a grade.
- Formative and diagnostic assessments are used to support learning and to provide timely feedback for students on their learning, and to teachers on their teaching.

Classroom Assessments

Assessments in the mathematics classroom are primarily formative and diagnostic in purpose. Classroom assessments include the questions teachers asked during lessons, the homework assigned to students, and class tests given at different times of the academic year. For these assessments to be formative, feedback to students is important. Students should use the feedback from these assessments to understand where they are in their learning and how to improve their learning.

GCE A-Level National Examination

Students will take the national examination in their final year. The national examination is a summative assessment that measures the level of attainment of the outcomes stated in the syllabuses.

The national examination code for the paper is **8865**. The examination syllabus can be found in the SEAB website. Important information about the examination is reproduced here.

Assessment Objectives (AO)

The assessment will test students' abilities to:

AO1 Understand and apply mathematical concepts and skills in a variety of problems, including those that may be set in unfamiliar contexts, or require integration of concepts and skills from more than one topic.
AO2 Formulate real-world problems mathematically, solve the mathematical problems, interpret and evaluate the mathematical solutions in the context of the problems.
AO3 Reason and communicate mathematically through making deductions and writing mathematical explanations and arguments.

The examinations will be based on the topic/sub-topic and content list on page 8 – 13. Knowledge of *O-level Mathematics* is assumed.

Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics.

While problems may be set based in contexts, no assumptions will be made about the knowledge of the contexts. All information will be self-contained within the problem.

Scheme of Examination Papers

There will be one 3-hour paper marked out of 100 as follows:

Section A (Pure Mathematics – 40 marks) will consist of about 5 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.
Section B (Probability and Statistics – 60 marks) will consist of 6 to 8 questions of different lengths and marks based on the Probability and Statistics section of the syllabus.

There will be at least two questions, with at least one in each section, on application of Mathematics in real-world contexts, including those from business and the social sciences. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Students will be expected to answer **all** questions.

Use of a graphing calculator (GC)

The use of an approved GC *without* computer algebra system will be expected. The examination papers will be set with the assumption that students will have access to GC. As a general rule, unsupported answers obtained from GC are allowed unless the question states otherwise. Where unsupported answers from GC are not allowed, students are required to present the mathematical steps using mathematical notations and not calculator commands. For questions where graphs are used to find a solution, students should sketch these graphs as part of their answers. Incorrect answers without working will receive no marks. However, if there is written evidence of using GC correctly, method marks may be awarded.

Students should be aware that there are limitations inherent in GC. For example, answers obtained by tracing along a graph to find roots of an equation may not produce the required accuracy.

List of formulae and statistical tables

Students will be provided in the examination with a list of formulae and statistical tables.

Mathematical notation

A list of mathematical notation is available at the SEAB website.

5. USEFUL REFERENCE BOOKS

Pure Mathematics

- Anton, H., Bivens, I. & Davis S. (2001). Calculus brief edition. (7th Ed.). Wiley.
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*The list is by no means exhaustive as they provide some samples that students can refer to. There are other reference books that students can use as well.