MATHEMATICS
SYLLABUS
Secondary One to Four
Normal (Technical) Course

Implementation starting with
2013 Secondary One Cohort

Ministry of Education
SINGAPORE

Learning Mathematics

A 21st Century Necessity

Learning mathematics is a key fundamental in every education system that aims to prepare its citizens for a productive life in the 21st century.

As a nation, the development of a highly-skilled and well-educated manpower is critical to support an innovation- and technology-driven economy. A strong grounding in mathematics and a talent pool in mathematics are essential to support the wide range of value-added economic activities and innovations. Many countries are paying attention to the quality of their mathematics education. The growing interest in TIMSS and PISA speaks of the global interest and importance placed on mathematics education.

At the individual level, mathematics underpins many aspects of our everyday activities, from making sense of information in the newspaper to making informed decisions about personal finances. It supports learning in many fields of study, whether it is in the sciences or in business. A good understanding of basic mathematics is essential wherever calculations, measurements, graphical interpretations and statistical analysis are necessary. The learning of mathematics also provides an excellent vehicle to train the mind, and to develop the capacity to think logically, abstractly, critically and creatively. These are important 21st century competencies that we must imbue in our students, so that they can lead a productive life and be life-long learners.

Students have different starting points. Not all will have the same interests and natural abilities to learn mathematics. Some will find it enjoyable; others will find it challenging. Some will find the theorems and results intriguing; others will find the formulae and rules bewildering. It is therefore important for the mathematics curriculum to provide differentiated pathways and choices to support every learner in order to maximise their potential. The curriculum must engage the 21st century learners, who are digital natives comfortable with the use of technologies and who work and think differently. The learning of mathematics must take into cognisance the new generation of learners, the innovations in pedagogies as well as the affordances of technologies.

It is the goal of the national mathematics curriculum to ensure that all students will achieve a level of mastery of mathematics that will serve them well in their lives, and for those who have the interest and ability, to pursue mathematics at the highest possible level. Mathematics is an important subject in our national curriculum. Students begin to learn mathematics from the day they start formal schooling, and minimally up to the end of secondary education. This gives every child at least 10 years of meaningful mathematics education.

About this document

This document provides an overview of the curriculum. It explains the design of the curriculum from the primary to the pre-university level, and provides details of the N(T)-Level Mathematics syllabus, including the aims, content, outcomes and the approaches to teaching and learning.

This document comprises 5 chapters as described below.

Chapter 1 provides an overview of the curriculum review, the goals and aims of the different syllabuses of the entire mathematics curriculum (primary to pre-university) as well as the syllabus design considerations across the levels.

Chapter 2 elaborates on the Mathematics Framework which centres around mathematical problem solving. The framework serves as a guide for mathematics teaching, learning and assessment across the levels.

Chapter 3 focuses on the process of teaching and learning so as to bring about engaged learning in mathematics. It highlights the principles of teaching and phases of learning as well as the learning experiences to influence the way teachers teach and students learn so that the aims of the curriculum can be met. The role of assessment and how it can be integrated to support learning in the classroom is also highlighted in this chapter.

Chapter 4 details the N(T)-Level Mathematics syllabuses in terms of its aims, syllabus organisation, mathematical processes, content and learning experiences.

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Background

Staying Relevant and Forward-Looking

As in all previous reviews, the 2010 full-term review aims to update the syllabuses so that they continue to meet the needs of our students, build a strong foundation in mathematics, and make improvement in the school mathematics education. It takes into consideration the analyses of students' performances in national examinations as well as international studies such as TIMSS and PISA. This review also takes on board the curriculum-wide recommendations from envisaging studies into the overall Singapore curriculum such as seeking a better balance between content and skills, creating opportunities to develop 21st century competencies, promoting self-directed and collaborative learning through ICT-based lessons, and developing assessment to support learning.

It is clear at the start of the review that there is more to be considered than just focusing on the content. While there is a need to constantly review what students learn, the changes in content will not be the key lever. In fact, little has been changed in the content as this has stabilised over the years. Instead, more focus has now been given to skills and competencies that will make a better 21st century learner – the process of learning becomes more important than just what is to be taught and remembered. The syllabuses are therefore written with the view that not only will it inform teachers on what to teach, it will also influence the way teachers teach and students learn. One key feature of this set of syllabuses is the explication of learning experiences, besides the learning outcomes. This gives guidance to teachers on the opportunities that students should be given as part of their learning. Ultimately, how students learn matters.

Curriculum review and design is ongoing work. The quality of the curriculum is as much in its design as it is in its implementation. Teachers, who are the frontline of curriculum delivery, must believe in the value of the changes. Support, resources and training will be provided to build capacity in our teachers. All these will be part of the continuous effort to deliver the best mathematics curriculum for the students.

The N(T)-Level Mathematics Syllabus will be implemented level by level starting from Secondary One in 2013. The implementation schedule is as follows:

Year	2013	2014	2015	2016
Level	Sec 1	Sec 2	Sec 3	Sec 4

This online syllabus document will be updated yearly according to the implementation schedule

Goals and Aims

Different Syllabuses, Different Aims

The overarching goal of the mathematics curriculum is to ensure that all students will achieve a level of mastery of mathematics that will serve them well in life, and for those who have the interest and ability, to pursue mathematics at the highest possible level.

The broad aims of mathematics education in Singapore are to enable students to:

- acquire and apply mathematical concepts and skills;
- develop cognitive and metacognitive skills through a mathematical approach to problem solving; and
- develop positive attitudes towards mathematics.

The mathematics curriculum comprises a set of syllabuses spanning 12 years, from primary to pre-university, and is compulsory up to the end of secondary education. Each syllabus has its own specific set of aims to guide the design and implementation of the syllabus. The aims also influence the choice of content, skills as well as contexts to meet the specific needs of the students at the given level or course. Each syllabus expands on the three broad aims of mathematics education differently to cater for the different needs and abilities of the students (see table of aims on the next page).

What does it mean to teachers?

Understanding the aims of the syllabus helps teachers stay focused on the larger outcomes of learning and guides teachers when they embark on the school-based curriculum innovations and customisations.

Overview of Aims Across the Levels

Primary
Laying a Strong Foundation

The Primary Mathematics syllabus aims to enable all students to:

- acquire mathematical concepts and skills for everyday use and for continuous learning in mathematics;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving; and
- build confidence and foster interest in mathematics.

Secondary
Building Up Strengths

The O- and N(A)-Level Mathematics syllabuses aim to enable all students to:

- acquire mathematical concepts and skills for continuous learning in mathematics and to support learning in other subjects;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving;
- connect ideas within mathematics and between mathematics and other subjects through applications of mathematics; and
- build confidence and foster interest in mathematics.

The O- and N(A)-Level Additional Mathematics syllabuses aim to enable students who have an aptitude and interest in mathematics to:

- acquire mathematical concepts and skills for higher studies in mathematics and to support learning in the other subjects, in particular, the sciences;
- develop thinking, reasoning and metacognitive skills through a mathematical approach to problem solving;
- connect ideas within mathematics and between mathematics and the sciences through applications of mathematics; and
- appreciate the abstract nature and power of mathematics.

The N(T)-Level Mathematics syllabus aims to enable students who are bound for post-secondary vocational education to:

- acquire mathematical concepts and skills for real life, to support learning in other subjects, and to prepare for vocational education;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving; and
- build confidence in using mathematics and appreciate its value in making informed decisions in real life.

Pre-University
Gearing Up for University Education

The H1 Mathematics syllabus aims to enable students who are interested in pursuing tertiary studies in business and the social sciences to:

- acquire mathematical concepts and skills to support their tertiary studies in business and the social sciences;
- develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem solving;
- connect ideas within mathematics and between mathematics and other disciplines through applications of mathematics; and
- appreciate the value of mathematics in making informed decisions in life.

The H2 Mathematics syllabus aims to enable students who are interested in pursuing tertiary studies in mathematics, sciences and engineering to:

- acquire mathematical concepts and skills to prepare for their tertiary studies in mathematics, sciences and engineering;
- develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem solving and the use of mathematics language;
- connect ideas within mathematics and between mathematics and other disciplines through applications of mathematics; and
- appreciate the beauty of mathematics and its value in making informed decisions in life.

The H3 Mathematics syllabus aims to enable students who have an aptitude and passion for mathematics to:

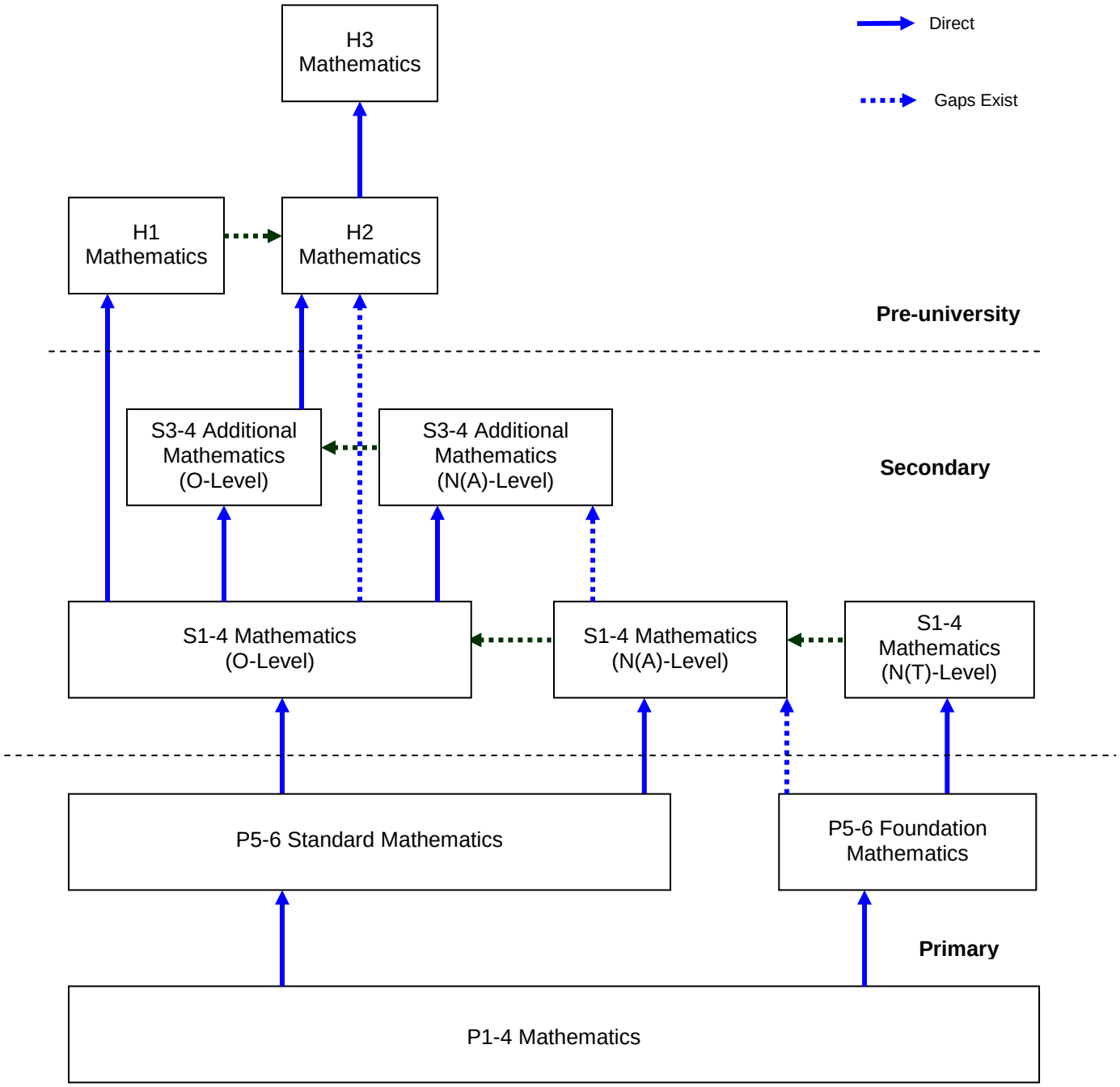
- acquire advanced mathematical concepts and skills to deepen their understanding of mathematics, and to widen the scope of applications of mathematics;
- develop rigorous habits of mind through mathematical reasoning and proof, creative mathematical problem solving, and use of mathematical models;
- connect ideas within mathematics at a higher level and between mathematics and other disciplines through applications of mathematics; and
- appreciate the beauty, rigour and abstraction of mathematics through mathematical proof and applications.

Syllabus Design

Spiral Curriculum, Connected Syllabuses

Mathematics is largely hierarchical in nature. Higher concepts and skills are built upon the more foundational ones and have to be learned in sequence. A spiral approach is adopted in the building up of content across the levels.

The mathematics curriculum consists of a set of connected syllabuses to cater to the different needs and abilities of students. This section gives an overview of the syllabuses and their connections so that teachers are better able to appreciate the mathematics curriculum as a whole.



The Primary Mathematics syllabus assumes no formal learning of mathematics. However, basic pre-numeracy skills such as matching, sorting and comparing are necessary in providing a good grounding for students to begin learning at Primary 1 (P1).

The P1-4 syllabus is common to all students. The P5-6 Standard Mathematics syllabus continues the development of the P1-4 syllabus whereas the P5-6 Foundation Mathematics syllabus re-visits some of the important concepts and skills in the P1-4 syllabus. The new concepts and skills introduced in Foundation Mathematics is a subset of the Standard Mathematics syllabus.

The O-Level Mathematics syllabus builds on the Standard Mathematics syllabus. The N(A)-Level Mathematics syllabus is a subset of O-Level Mathematics, except that it re-visits some of the topics in Standard Mathematics syllabus. The N(T)-Level Mathematics syllabus builds on the Foundation Mathematics syllabus.

The O-Level Additional Mathematics syllabus assumes knowledge of O-Level Mathematics content and includes more in-depth treatment of important topics. The N(A)-Level Additional Mathematics is a subset of O-Level Additional Mathematics. O-Level Additional Mathematics together with O-Level Mathematics content provide the prerequisite knowledge required for H2 Mathematics at the pre-university level.

At the pre-university level, mathematics is optional. The H1 Mathematics syllabus builds on the O-level Mathematics syllabus. H2 Mathematics assumes some of the O-Level Additional Mathematics content. H3 Mathematics is an extension of H2 Mathematics.

Flexibility and Choice

There are two mathematics syllabuses at the P5-6 level. Most students would offer Standard Mathematics and for students who need more time to learn, they could offer Foundation Mathematics.

There are five mathematics syllabuses in the secondary mathematics curriculum. O-Level Mathematics, N(A)-Level Mathematics and N(T)-Level Mathematics provide students from the respective courses the core mathematics knowledge and skills in the context of a broad-based education. The more mathematically able students from the N(A) course can choose to take O-Level Mathematics in four years instead of five years. Likewise, the more able N(T) course students can also offer N(A)-Level Mathematics. The variation in pace and syllabus adds to the flexibility and choice within the secondary mathematics curriculum. At the upper secondary level, students who have the interests and abilities in mathematics may choose to offer Additional Mathematics as an elective at the O-Level or N(A)-Level. This gives students with an inclination towards and interest in mathematics the opportunity to learn more mathematics that would prepare them well for courses of study that require higher mathematics.

For students who wish to study in the Engineering-type courses at the polytechnics, Additional Mathematics will be a good grounding. The N(A)-Level and N(T)-Level Mathematics syllabuses will prepare students well for ITE courses. Students who aspire to study Mathematics or mathematics-related courses at the universities could offer H2 Mathematics, and if possible, H3 Mathematics.

What does it mean to teachers?

Teachers need to have the big picture in mind so that they can better understand the role of each syllabus, the connection it makes with the next level and the dependency relationship between syllabuses. This enables teachers to better understand what they have to do at their level, as well as to plan and advise students in their learning of mathematics. For example, H2 Mathematics assumes some of the O-Level Additional Mathematics content but may be offered by students without Additional Mathematics background as long as effort is made to bridge the gap.

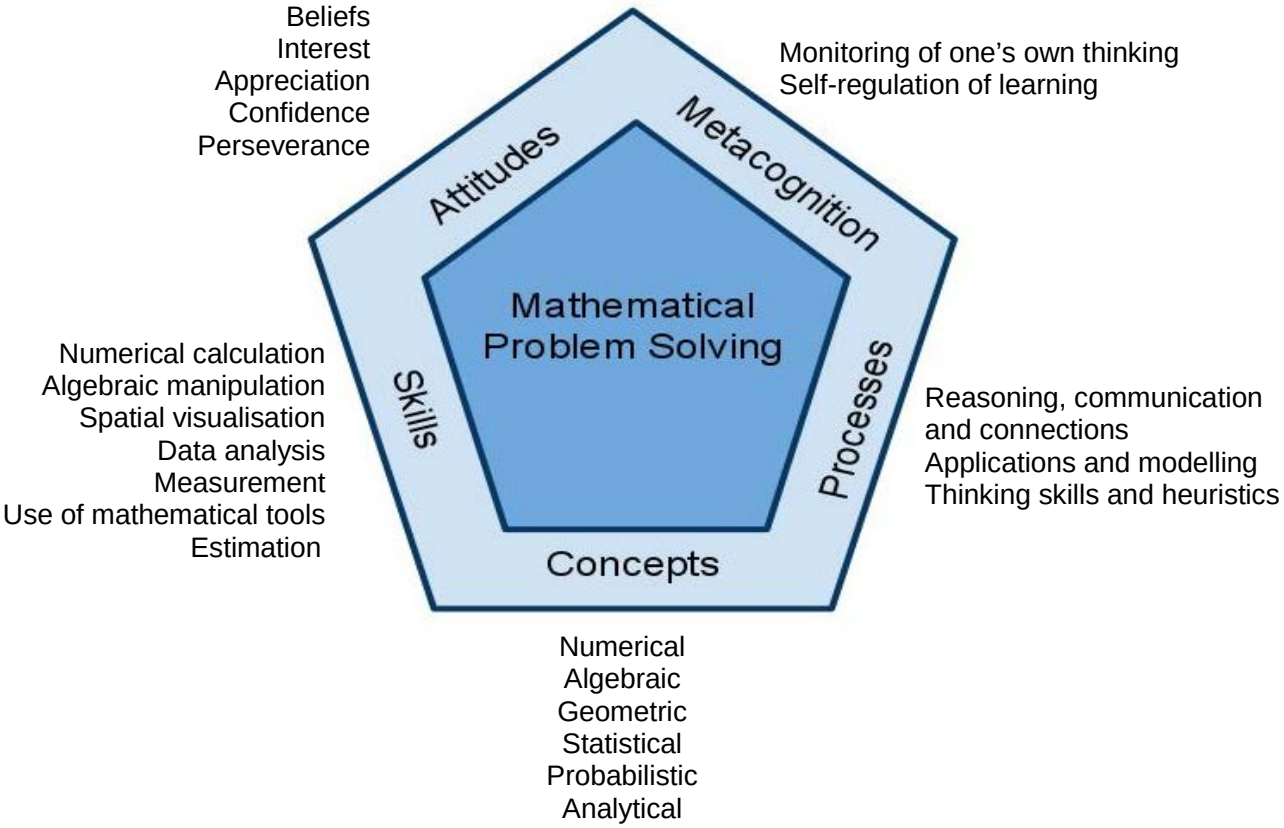
Chapter 2
Mathematics Framework

Problem Solving

Problem Solving

Concepts, Skills, Processes, Metacognition, Attitudes

The Mathematics Framework has been a feature of our mathematics curriculum since 1990, and is still relevant to date. The central focus of the framework is mathematical problem solving, that is, using mathematics to solve problems. The framework sets the direction for and provides guidance in the teaching, learning, and assessment of mathematics at all levels, from primary to pre-university. It reflects also the 21st century competencies[1].



The framework stresses *conceptual understanding*, *skills proficiency* and *mathematical processes*, and gives due emphasis to *attitudes* and *metacognition*. These five components are inter-related.

Concepts

Mathematical concepts can be broadly grouped into *numerical*, *algebraic*, *geometric*, *statistical*, *probabilistic*, and *analytical* concepts. These content categories are connected and interdependent. At different stages of learning and in different syllabuses, the breadth and depth of the content vary.

[1] Information on the MOE framework for 21st century competencies and student outcomes can be found on edumall2.0.

To develop a deep understanding of mathematical concepts, and to make sense of various mathematical ideas as well as their connections and applications, students should be exposed to a variety of learning experiences including hands-on activities and use of technological aids to help them relate abstract mathematical concepts with concrete experiences.

Skills

Mathematical skills refer to *numerical calculation*, *algebraic manipulation*, *spatial visualisation*, *data analysis*, *measurement*, *use of mathematical tools*, and *estimation*. The skills are specific to mathematics and are important in the learning and application of mathematics. In today's classroom, these skills also include the abilities to use spreadsheets and other software to learn and do mathematics.

To develop proficiencies in mathematics skills, students should have opportunities to use and practise the skills. These skills should be taught with an understanding of the underlying mathematical principles and not merely as procedures.

Processes

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. These include *reasoning, communication and connections*, *applications and modelling*, and *thinking skills and heuristics* that are important in mathematics and beyond.

In the context of mathematics, *reasoning, communication and connections* take on special meanings:

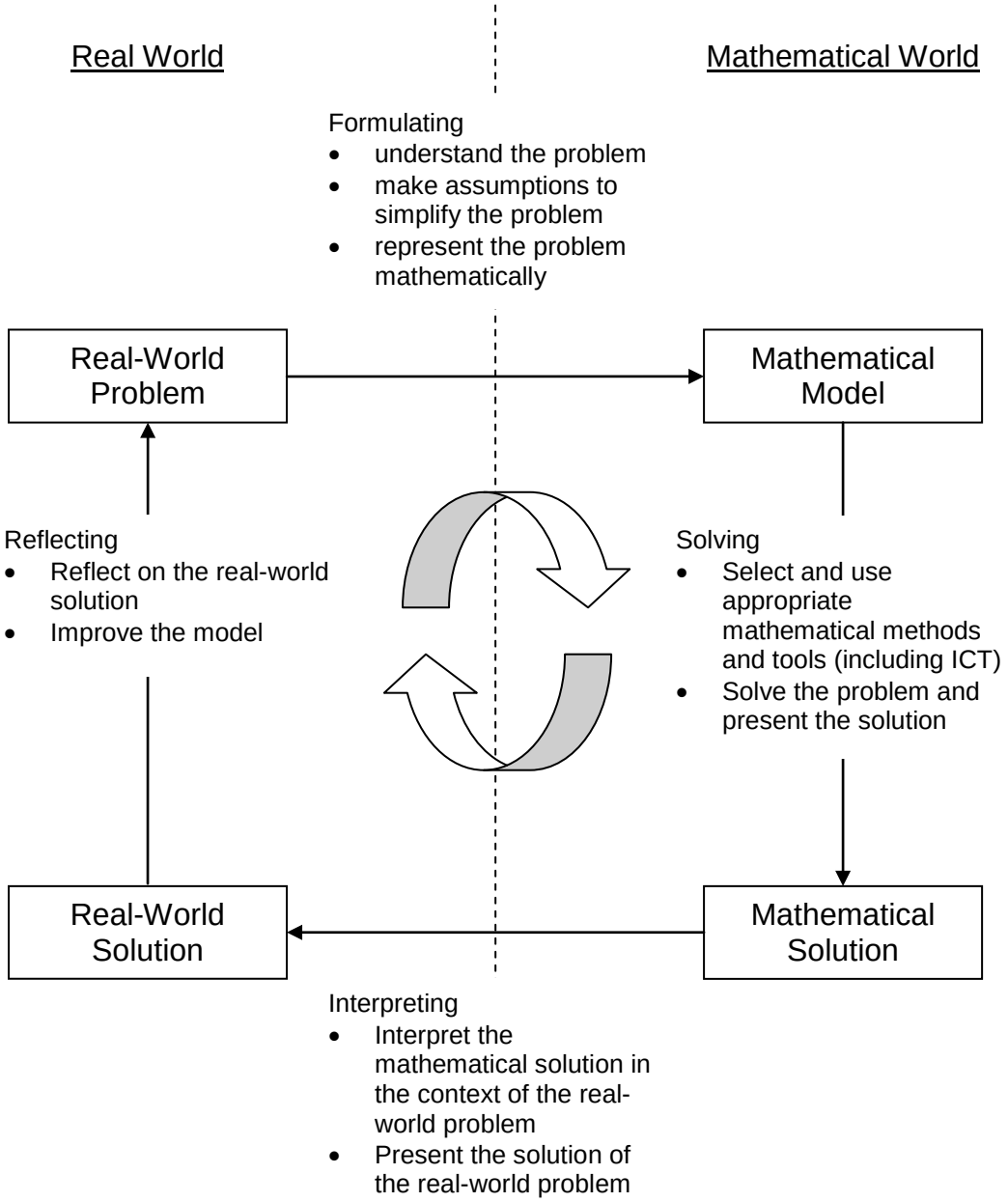
- Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments. It is a habit of mind that can be developed through application of mathematics in different contexts.
- Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically. It helps students develop their understanding of mathematics and sharpen their mathematical thinking.
- Connections refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and the real world. This helps students make sense of what they learn in mathematics.

Applications and modelling allow students to connect mathematics that they have learnt to the real world, enhance understanding of key mathematical concepts and methods as well as develop mathematical competencies. Students should have opportunities to apply mathematical problem-solving and reasoning skills to tackle a variety of problems, including open-ended and real-world problems. Mathematical modelling is the process of formulating and improving a mathematical model[2] to

[2] A mathematical model is a mathematical representation or idealisation of a real-world situation. It can be as complicated as a system of equations or as simple as a geometrical figure. As the word "model" suggests, it shares characteristics of the real-world situation that it seeks to represent.

represent and solve real-world problems. Through mathematical modelling, students learn to deal with ambiguity, make connections, select and apply appropriate mathematical concepts and skills, identify assumptions and reflect on the solutions to real-world problems, and make informed decisions based on given or collected data.

Mathematical Modelling Process (version 2010)



Thinking skills and heuristics are essential for mathematical problem solving. Thinking skills are skills that can be used in a thinking process, such as classifying, comparing, analysing parts and whole, identifying patterns and relationships, induction, deduction, generalising, and spatial visualisation. Heuristics are general

rules of thumb of what students can do to tackle a problem when the solution to the problem is not obvious. These include using a representation (e.g., drawing a diagram, tabulating), making a guess (e.g., trial and error/guess and check, making a supposition), walking through the process (e.g., acting it out, working backwards) and changing the problem (e.g., simplifying the problem, considering special cases).

Metacognition

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning.

To develop metacognitive awareness and strategies, and know when and how to use the strategies, students should have opportunities to solve non-routine and open-ended problems, to discuss their solutions, to think aloud and reflect on what they are doing, and to keep track of how things are going and make changes when necessary.

Attitudes

Attitudes refer to the affective aspects of mathematics learning such as:

- beliefs about mathematics and its usefulness;
- interest and enjoyment in learning mathematics;
- appreciation of the beauty and power of mathematics;
- confidence in using mathematics; and
- perseverance in solving a problem.

Students' attitudes towards mathematics are shaped by their learning experiences. Making the learning of mathematics fun, meaningful and relevant goes a long way to inculcating positive attitudes towards the subject. Care and attention should be given to the design of the learning activities to build confidence in and develop appreciation for the subject. Above all, students' beliefs can influence their attitudes in learning, especially in student-centred learning where students are encouraged to take on more responsibility for their own learning.

What does it mean to teachers?

The five components of the Mathematics Framework are integral parts of mathematics learning and problem solving. The intent of the framework is to help teachers focus on these components in their teaching practice so as to provide a more engaging, student-centred, and technology-enabled learning environment, and to promote greater diversity and creativity in learning.

Chapter 3
Teaching, Learning and Assessment

Learning Experiences
Teaching and Learning
Assessment in the Classroom

Learning Experiences

It matters how students learn

Learning mathematics is more than just learning concepts and skills. Equally important are the cognitive and metacognitive process skills. These processes are learned through carefully constructed learning experiences. For example, to encourage students to be inquisitive, the learning experiences must include opportunities where students discover mathematical results on their own. To support the development of collaborative and communication skills, students must be given opportunities to work together on a problem and present their ideas using appropriate mathematical language and methods. To develop habits of self-directed learning, students must be given opportunities to set learning goals and work towards them purposefully. A classroom, rich with these opportunities, will provide the platform for students to develop these 21st century competencies.

Learning experiences are stated in the mathematics syllabuses to influence the ways teachers teach and students learn so that the curriculum objectives can be achieved. These statements expressed in the form "students should have opportunities to …" remind teachers of the student-centric nature of these experiences. They describe actions that students will perform and activities that students will go through, with the opportunities created and guidance rendered by teachers. The descriptions are sufficiently specific to provide guidance yet broad enough to give flexibility to the teachers.

For each topic, the learning experiences focus on the mathematical processes and skills that are integral parts of learning of that topic. There are also generic learning experiences that focus on the development of good learning habits and skills such as:

Students should have opportunities to:

- take notes and organise information meaningfully;
- practise basic mathematical skills to achieve mastery;
- use feedback from assessment to improve learning;
- solve novel problems using a repertoire of heuristics;
- discuss, articulate and explain ideas to develop reasoning skills; and
- carry out a modelling project.

These learning experiences, whether they are topical or generic, are not exhaustive. Teachers are encouraged to do more to make learning meaningful and effective.

Teaching and Learning

Principles of Teaching and Phases of Learning

This section outlines three principles of mathematics teaching and the three phases of mathematics learning in the classrooms.

Principles of Teaching

Principle 1
Teaching is for learning; learning is for understanding; understanding is for reasoning and applying and, ultimately problem solving.

Teaching is an interactive process that is focused on students' learning. In this process, teachers use a range of teaching approaches to engage students in learning; students provide teachers with feedback on what they have learnt through assessment; and teachers in turn provide feedback to students and make decisions about instructions to improve learning.

The learning of mathematics should focus on understanding, not just recall of facts or reproduction of procedures. Understanding is necessary for deep learning and mastery. Only with understanding can students be able to reason mathematically and apply mathematics to solve a range of problems. After all, problem solving is the focus of the mathematics curriculum.

Principle 2
Teaching should build on students' knowledge; take cognizance of students' interests and experiences; and engage them in active and reflective learning.

Mathematics is a hierarchical subject. Without understanding of pre-requisite knowledge, foundation will be weak and learning will be shallow. It is important for teachers to check on students' understanding before introducing new concepts and skills.

Teachers need to be aware of their students' interests and abilities so as to develop learning tasks that are stimulating and challenging. This is important in order to engage students in active and reflective learning where students participate and take ownership of the learning.

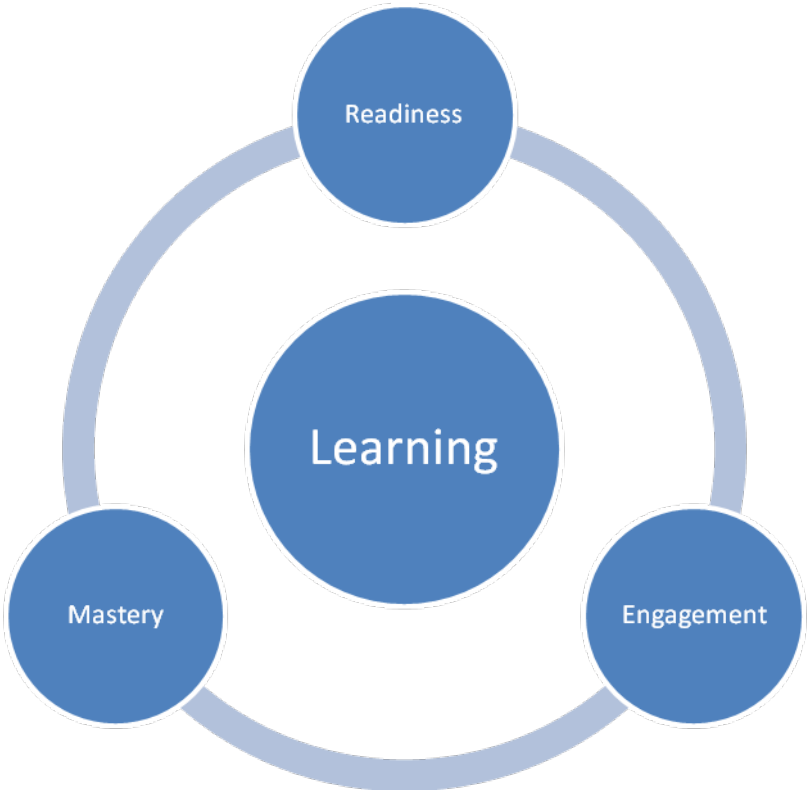
Principle 3
Teaching should connect learning to the real world, harness ICT tools and emphasise 21st century competencies.

There are many applications of mathematics in the real world. Students should have an understanding and appreciation of these applications and how mathematics is used to model and solve problems in real-world contexts. In this way, students will see the meaning and relevance of mathematics.

Teachers should consider the affordances of ICT to help students learn. ICT tools can help students understand mathematical concepts through visualisations, simulations and representations. They can also support exploration and experimentation and extend the range of problems accessible to students. The ability to use ICT tools is part of the 21st century competencies. It is also important to design learning in ways that promote the development of other 21st century competencies such as working collaboratively and thinking critically about the mathematical solution.

Phases of Learning

Effective instruction of a unit typically involves three phases of learning: *Readiness*, *Engagement* and *Mastery*.



Phase 1 - *Readiness*

Student readiness to learn is vital to learning success. In the readiness phase of learning, teachers prepare students so that they are ready to learn. This requires considerations of *prior knowledge*, *motivating contexts*, and *learning environment*.

- ***Prior Knowledge***
  For students to be ready to learn, teachers need to know students' prior knowledge in relation to the new learning. This requires knowing whether students have the pre-requisite concepts and skills. Some form of diagnostic assessment is necessary to check that students are ready to learn.

- ***Motivating Contexts***
  For students to be ready to learn, teachers need to provide motivating contexts for learning. These contexts should be developmentally appropriate. For example, younger students may like contexts such as stories and songs, and play-based activities such as games, whereas older students may appreciate contexts related to everyday life so that they can see the relevance and meaningfulness of mathematics. For the more advanced students, applications in other disciplines can serve as motivation for learning.

- ***Learning Environment***
  Shared rules help promote respectful and emotionally-safe interactions between teacher and students and among students that are necessary for productive and purposeful learning. Established procedures for organising students and managing resources will also facilitate a smooth start and transitions during lessons.

Phase 2 - *Engagement*

This is the main phase of learning where teachers use a repertoire of pedagogies to engage students in learning new concepts and skills. Three pedagogical approaches form the spine that supports most of the mathematics instruction in the classroom. They are not mutually exclusive and could be used in different parts of a lesson or unit. For example, the lesson or unit could start with an activity, followed by teacher-led inquiry and end with direct instruction.

- ***Activity-based Learning***
  This approach is about learning by doing. It is particularly effective for teaching mathematical concepts and skills at primary and lower secondary levels, but are also effective at higher levels. Students engage in activities to explore and learn mathematical concepts and skills, individually or in groups. They could use manipulatives or other resources to construct meanings and understandings. From concrete manipulatives and experiences, students are guided to uncover abstract mathematical concepts or results.

  For example, in teaching the area of special quadrilaterals, the teacher designs an activity-based lesson in which students use paper cut-outs to relate the area of a triangle/parallelogram to that of a rectangle, and the area of a trapezium to that of a parallelogram. The hands on approach helps students visualise and develop understanding of how the formulae for areas of the various shapes are obtained, leading to deeper conceptual learning and better retention of learning. It is important that the teacher structures the activity to allow for meaningful discussion and to help students see the connection between the task and the mathematical concepts and skills to be learnt.

- ***Teacher-directed Inquiry***
  This approach is about learning through guided inquiry. Instead of giving the answers, teachers lead students to explore, investigate and find answers on their own. Students learn to focus on specific questions and ideas and are engaged in communicating, explaining and reflecting on their answers. They also learn to pose questions, process information and data and seek appropriate methods and solutions. This enhances the development of mathematical processes and 21st century competencies.

  For example, in teaching the uses and purposes, and advantages and disadvantages of different forms of statistical representations, the teacher leverages the affordances of ICT to compare graphs and investigate how mean, mode and median are affected by changes in data values. Through teacher-directed inquiry, the teacher elicits student observations and facilitates discussions about relationships between data values and their representations, and data analyses and interpretations. Students are encouraged to communicate their ideas using appropriate mathematical language. The teacher also assesses students' understanding, identifies gaps and misconceptions, and reinforces key concepts through continuous interactive assessments.

- ***Direct Instruction***
  This approach is about explicit teaching. Teachers introduce, explain and demonstrate new concepts and skill. Direct instruction is most effective when students are told what they will be learning and what they are expected to be able to do. This helps them focus on the learning goals. Teachers draw connections, pose questions, emphasise key concepts, and role-model thinking. Holding students' attention is critical. Stimuli such as videos, graphic images, real-world contexts, and even humour, aid in maintaining a high level of attention.

  For example, in teaching percentage in the context of discounts, taxation, etc, the teacher states the objectives clearly and outlines the progression of the learning that they will be going through. He scaffolds students' understanding of the problem through careful questioning as he introduces and explains each concept and skill. To help students develop strong conceptual understanding, the teacher models thinking aloud, making explicit the thought process that finding a percentage part of a quantity is equivalent to finding a fractional part of the quantity. The teacher asks probing questions to elicit students' understanding, corrects misconceptions that surface, and guides them in the process of solving the problems. During lesson closure, the teacher reviews the key learning points of the lesson to reinforce the learning.

Phase 3 - *Mastery*

This is the final phase of learning where teachers help students consolidate and extend their learning. The mastery approaches include:

- ***Motivated Practice***
  Students need practice to achieve mastery. Practice can be motivating and fun. Practice must include repetition and variation to achieve proficiency and flexibility. Structuring practice in the form of games is one good strategy to make practice motivating and fun, while allowing for repetition and variation. There should be a range of activities, from simple recall of facts to application of concepts.

- ***Reflective Review***
  It is important that students consolidate and deepen their learning through tasks that allow them to reflect on their learning. This is a good habit that needs to be cultivated from an early age and it supports the development of metacognition. Summarising their learning using concept maps, writing journals to reflect on their learning and making connections between mathematical ideas and between mathematics and other subjects should be encouraged. Sharing such reflections through blogs makes learning social.

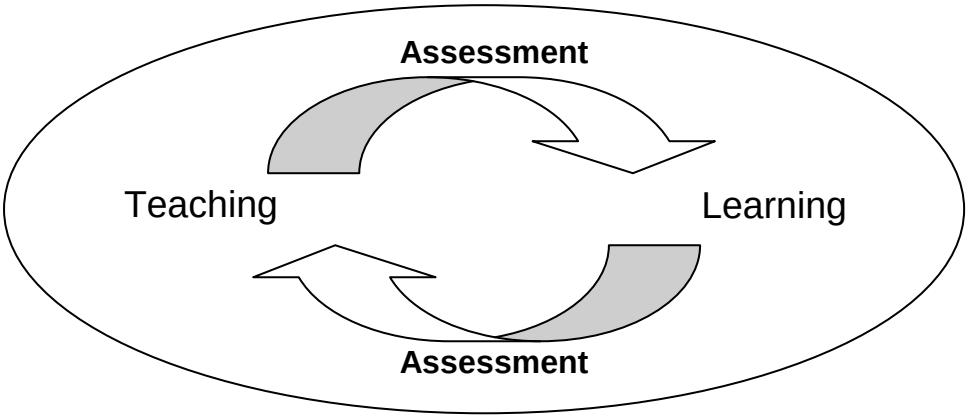
- ***Extended Learning***
  Students who are mathematically inclined should have opportunities to extend their learning. These can be in the form of more challenging tasks that stretch their thinking and deepen their understanding.

Assessment in the Classroom

Supporting Teaching and Learning in Mathematics

Role of assessment

Assessment is an integral part of the interactive process of teaching and learning, as illustrated in the diagram below. It is an ongoing process by which teachers gather information about students' learning to inform and support teaching. An important product of assessment is feedback. Feedback must be timely and rich. It must inform students where they are in their learning and what they need to do to improve their learning. It must also inform teachers what they need to do to address learning gaps and how to improve their instruction.



Range of assessment

Assessments can be broadly classified as summative, formative, and diagnostic.

- Summative assessments, such as tests and examinations, measure what students have learned. Teachers usually report the assessment result as a score or a grade.

- Formative and diagnostic assessments are used as assessment for learning to provide timely feedback to students on their learning, and to teachers on their teaching.

Assessment in the classroom should focus on helping students improve their learning. Therefore, they are primarily formative and diagnostic in purpose.

Though teachers are comfortable with the use of traditional pen-and-paper tests to find out how much students know and can do, there is value in exploring a wider variety of assessment strategies. These strategies allow teachers to gather information that is not easily available through traditional methods of assessment,

but are nevertheless valuable in supporting learning. Ultimately, the choice of assessment strategies must be guided by its purpose, that is, it must be fit-for-purpose.

Integrating assessment with instruction

It is important that teachers know what and when to assess student learning, and how to embed the assessment in the learning process. Assessment can be integrated into classroom discourse and activities using different assessment strategies. For example, teachers may watch students solve problems and get them to explain their strategies. Teachers may also engage students in assessing their own work and reflecting on their own learning and how to improve it. Both moment-by-moment assessment and planned assessment should be considered.

Effective questioning can scaffold learning and probe understanding. It creates teachable moments for teachers to correct a misconception, reinforce a point or expand on an idea. The questions can be open-ended to encourage students to consider alternative approaches. Sufficient wait-time is necessary so that students can formulate their thoughts, communicate and share their ideas, and hear the ideas of others. In the process, students learn to articulate their thinking and deepen their understanding, and develop confidence in talking about mathematics and using it. Teachers can assess students' thinking and understanding, and provide useful feedback to improve their learning.

Teachers can integrate performance assessments into the instructional process to provide additional learning experiences for students. This type of assessment requires students to apply their knowledge and skills in context, and the focus is on mathematical processes rather than on mathematics content. A rubric is useful to show teachers what to look for in students' work, but more importantly, it shows what is expected of students in terms of processes and quality of work. The rubric also provides a structured means of giving qualitative feedback. Teachers may allow students to assess their own performances so that they can reflect on their work and make improvements.

Assessment for learning calls for new ways of assessment in the classroom. It involves a change in teachers' roles and in the expectations of students. By integrating assessment and instruction, students will be more engaged in and will take greater ownership of their learning.

Chapter 4
N(T)-Level Mathematics Syllabus

Aims of Syllabus
Syllabus Organisation
Content and Learning Experiences
Mathematical Processes

Aims of Syllabus

The N(T)-Level Mathematics syllabus aims to enable students who are bound for post-secondary vocational education to:

- acquire mathematical concepts and skills for real life, to support learning in other subjects and to prepare for vocational education;
- develop thinking, reasoning, communication, application and metacognitive skills through a mathematical approach to problem solving; and
- build confidence in using mathematics and appreciate its value in making informed decisions in real life.

Syllabus Organisation

The N(T)-Level Mathematics syllabus is organised along 3 content strands, 'Number and Algebra', 'Geometry and Measurement', and 'Statistics and Probability' and a context strand 'Real-World Contexts', with a listing of mathematical processes that cut across the 4 strands.

<table>
<tr><td colspan="4">3 Content + 1 Context + 1 Process Strand</td></tr>
<tr><td>Number and Algebra</td><td>Geometry and Measurement</td><td>Statistics and Probability</td><td>Real-world Contexts</td></tr>
<tr><td colspan="4">Mathematical Processes</td></tr>
</table>

The write-up for each strand includes:

- a brief statement about the importance of the strand;
- what students will do at the secondary level; and
- how teaching and learning should be like for the strand.

Content by Strand

Strand: Number and Algebra

Number and algebra is the basic building block of school mathematics. A good understanding of numbers – their meanings, properties and operations provides the foundation for the learning of mathematics. Basic algebra concepts and skills and algebraic thinking are essential for problem solving and these should be developed progressively.

In the N(T) course, students build on and revisit their knowledge of whole numbers, fractions and decimals and learn about negative numbers and integers. They learn the meaning of these numbers and how they are used in different real world situations. Proficiency in computation - including good estimation and approximation skills and an awareness of errors and accuracy in computations are important. Word problems, involving the full range of numbers and relationships such as ratio and proportion, percentages, rate and speed provide an avenue for students to apply their learning. The habit of checking answers, both for accuracy as well as for reasonableness, continues to be emphasised.

In algebra, students begin using letters to represent numbers and quantitative relationships, generalise number sequences, manipulate simple algebraic expressions (simplification, expansion and factorisation), and solve equations. They learn the concept of function as a relationship between two variables e.g. direct and inverse proportions can be regarded as functional relationships. They also learn about linear functions and quadratic functions – their graphs, properties and applications.

The *Concrete – Pictorial – Abstract* approach is particularly relevant for this strand. The use of manipulatives and software helps students make sense of the meanings of operations including algebraic manipulations and connection between different representations including numerical (tabular), graphical and symbolic representations of functions. The model drawing method, which is used extensively at the primary level, should be used to help students formulate algebraic equations to solve problems at the secondary level in the early stage of learning algebra, building a bridge between arithmetic and algebraic manipulations. Even though calculators are allowed, it is important that students master the basic computational skills first so that they can check their answers by estimating.

No.	Topic/Sub-topics	Content
NUMBER AND ALGEBRA		
N1	**Numbers and their operations**	• negative numbers and primes (exclude prime factorisation) • integers and their four operations • four operations on fractions and decimals (including negative fractions and decimals) • calculations with calculator, including squares, cubes, square roots and cube roots • representation and ordering of numbers on the number line • use of <, >, ≤, ≥ • rounding off numbers to a required number of decimal places or significant figures • estimating the results of computation • use of index notation for integer powers • use of standard form $A \times 10^n$, where n is an integer, and $1 \leq A < 10$
N2	**Ratio and proportion**	• comparison between two or more quantities by ratio • dividing a quantity in a given ratio • ratios involving fractions and decimals • equivalent ratios • writing a ratio in its simplest form • direct and inverse proportion • map scales (distance and area) •
N3	**Percentage**	• expressing percentage as a fraction or decimal • finding the whole given a percentage part • expressing one quantity as a percentage of another • comparing two quantities by percentage • percentages greater than 100% • finding one quantity given the percentage and the other quantity • increasing/decreasing a quantity by a given percentage • finding percentage increase/decrease
N4	**Rate and speed**	• rates and average rates (including the concepts of speed and average speed) • conversion of units (e.g. km/h to m/s)
N5	**Algebraic expressions and formulae**	• using letters to represent numbers • interpreting notations: * ab as $a \times b$ * $\frac{a}{b}$ as $a \div b$ or $a \times \frac{1}{b}$ * a^2 as $a \times a$, a^3 as $a \times a \times a$, a^2b as $a \times a \times b$, . . . * $3y$ as $y + y + y$ or $3 \times y$ * $3(x+y)$ as $3 \times (x+y)$ * $\frac{3+y}{5}$ as $(3+y) \div 5$ or $\frac{1}{5} \times (3+y)$ • evaluation of algebraic expressions and formulae • translation of simple real-world situations into algebraic expressions • recognising and representing number sequences (include finding

No.	Topic/Sub-topics	Content
NUMBER AND ALGEBRA		
		an algebraic expression for the *n*th term for simple cases such as n+3, 2n+1) • addition and subtraction of linear expressions • simplification of linear expressions, e.g. * $-2(3x-5)+4x$ * $\frac{2x}{3}-\frac{3(x-5)}{2}$ • expansion of the product of two linear expressions • multiplication and division of simple algebraic fractions, e.g. * $\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ * $\frac{3a}{4}\div\frac{9a^2}{10}$ • changing the subject of a simple formula • finding the value of an unknown quantity in a given formula • factorisation of linear expressions of the form $ax+kay$ • Factorisation of quadratic expressions of the form x^2+px+q Exclude: • use of * $(a\pm b)^2=a^2\pm 2ab+b^2$ * $a^2-b^2=(a+b)(a-b)$ • addition and subtraction of algebraic fractions such as $\frac{1}{x}+\frac{1}{x-1}$
N6	**Functions and graphs**	• cartesian coordinates in two dimensions • graph of a set of ordered pairs as a representation of a relationship between two variables • linear functions $y=ax+b$ and quadratic functions $y=ax^2+bx+c$ • graphs of linear functions • the gradient of a linear graph as the ratio of the vertical change to the horizontal change (positive and negative gradients) • graphs of quadratic functions and their properties * positive or negative coefficient of x^2 * maximum and minimum points * symmetry
N7	**Solutions of Equations**	• solving linear equations in one variable • solving simple fractional equations that can be reduced to linear equations, e.g. * $\frac{x}{3}+\frac{x-2}{4}=3$ * $\frac{3}{x-2}=6$ • graphs of linear equations in two variables ($ax+by=c$) • solving simultaneous linear equations in two variables by

No.	Topic/Sub-topics	Content
NUMBER AND ALGEBRA		
		∗ substitution and elimination methods ∗ graphical method • solving quadratic equations in one variable by use of formula[3] • formulating a linear equation in one variable, a quadratic equation in one variable, or a pair of linear equations in two variables to solve problems

[3] For the GCE Exam, the formula for solving quadratic equations will be provided.

Strand: Geometry and Measurement

Geometry is the study of points, lines, angles, figures and their properties and relationships. It develops spatial visualisation and reasoning skills that are critical in solving geometry problems in the real world. *Measurement* involves finding length, mass, area, volume, time etc and units (e.g. cm, km, kg, minutes etc) are attached to numbers in order to make sense of the quantities. *Measurement* and *Geometry* have a common interest in shapes and figures that serves as models for real objects.

In the N(T) course, students extend their understanding of basic shapes and figures and their special properties to study a wider range of shapes and figures (e.g. trapezium and parallelogram), their properties and relationships to each other. They discover and learn about results (e.g. angles formed by two parallel lines and a transversal, Pythagoras' theorem) that are not immediately obvious but require some level of reasoning.

Students learn the trigonometry ratios and the applications of these ratios in calculation and measurement. They expand on the knowledge of formulae of length, area and volume for a wider range of shapes and figures (e.g. surface area and volume of a sphere).

The learning of *Geometry* at this stage should adopt an *intuitive and experimental* approach. This approach is based on van Hiele's theory of geometry learning which advocates exploration and discovery through hands-on activities. The use of dynamic geometry software is particularly relevant as it allows students to construct and manipulate geometric objects (points, lines, triangles, etc.) and to explore geometric properties and relationships. These activities should lead to discussion and justification of conclusions by students as part of the learning.

The learning of *Measurement* should involve a wide range of problems, including those in real-world contexts. This helps to reinforce the understanding of mathematical results, theorems and formulae, as well as concepts such as similarity and congruence.

No.	Topic/Sub-topics	Content
GEOMETRY AND MEASUREMENT		
G1	**Angles, triangles and quadrilaterals**	• right, acute, obtuse and reflex angles • vertically opposite angles, angles on a straight line and angles at a point • angles formed by two parallel lines and a transversal: corresponding angles, alternate angles, interior angles • properties of triangles and special quadrilaterals • properties of perpendicular bisectors of line segments and angle bisectors • construction of simple geometrical figures from given data (including perpendicular bisectors and angle bisectors) using compasses, rulers, set squares and protractors where appropriate

G2	**Symmetry, congruence and similarity**	• line and rotational symmetry of plane figures
		• lines of symmetry
		• order of rotational symmetry
		• congruent and similar figures
		• properties of similar triangles and quadrilaterals:
		  * corresponding angles are equal
		  * corresponding sides are proportional

G3	**Pythagoras' theorem and trigonometry**	• use of Pythagoras' theorem • determining whether a triangle is right-angled given the lengths of three sides • use of trigonometric ratios (sine, cosine and tangent) of acute angles to calculate unknown sides and angles in right-angled triangles (including problems involving angles of elevation and depression)
G4	**Mensuration**	• area of triangle as $\frac{1}{2}$ × base × height • area and circumference of circle • area of parallelogram and trapezium • problems involving perimeter and area of composite plane figures • visualising and sketching cube, cuboid, prism, cylinder, pyramid, cone and sphere (including use of nets to visualise the surface area of these solids, where applicable) • volume and surface area of cube, cuboid, prism, cylinder, pyramid, cone and sphere[4] • conversion between cm^2 and m^2, and between cm^3 and m^3 • problems involving volume and surface area of composite solids • arc length and sector area as fractions of the circumference and area of a circle

[4] For the GCE Exam, the following formula will be provided: curved surface area of a cone, surface area of a sphere, volume of pyramid, cone and sphere.

Strand: Statistics and Probability

Statistics provides the methods and tools to summarise, represent, analyse and interpret data so that useful information can be derived for making decisions, understanding a situation, etc. *Probability* provides the means to understand chance, randomness and uncertainty that pervades our everyday life. A good understanding of statistics and probability enhances a person's understanding of the world of information around them.

In the N(T) course, students learn a wider range of statistical diagrams and representations and extend beyond a singular measure of data (i.e. average) to other measures of central tendency (e.g. mean, mode, median) and spread (e.g. inter-quartile range). The focus is on the descriptive aspects of statistics with analysis, interpretation and making simple decisions from the data. Students learn to read statistics in materials critically such as those appearing in public domains (e.g. in reports or advertisements in the mass media).

For probability, students learn the concept as a measure of chance of events that are random or uncertain. They learn to compute probability for a single event involving counting of equally likely outcomes.

The learning of statistics and probability should involve hands-on experiences where students have to collect data (e.g. for a simple survey) and conduct experiments (e.g. rolling a dice, tossing a coin, playing with cards). The use of ICT tools such as a spreadsheet to generate statistical diagrams to represent the data or to simulate and generate the outcomes of random events should be part of the learning experience of the students. An exploratory approach using authentic materials that encourages inquiry about the data and the phenomenon is advocated.

No.	Topic/Sub-topics	Content
STATISTICS AND PROBABILITY		

S1	**Data analysis**	• analysis and interpretation of:
		  ∗ tables
		  ∗ bar graphs
		  ∗ pictograms
		  ∗ line graphs
		  ∗ pie charts
		  ∗ dot diagrams
		  ∗ histograms with equal intervals
		• purposes and use, advantages and disadvantages of the different forms of statistical representations
		• purposes and use of mean, mode and median
		• calculation of the mean, mode and median for a set of ungrouped data
		• percentiles, quartiles, range and interquartile range
		• analysis and interpretation of cumulative frequency diagrams

S2	**Probability**	• probability as a measure of chance • probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability)

Strand: Real-world Contexts

For learning to be meaningful and relevant, it is important for students to apply what they learn in mathematics to real-world contexts.

In the N(T) course, special attention is given to solving problems in real-world contexts. This is in line with the functional approach to the teaching of mathematics for these students. Working with real-world problems enhances the students' mathematical understanding by making connections between mathematics and real-world situations. These situations may arise from personal finance, newspaper advertisements, and media reports etc which students may encounter in their life and where a good understanding of mathematics will help them make informed decisions. Students will need to understand the contexts and distil the mathematics in the contexts to solve the problems. They would have opportunities to make connections across strands and the different representations to solve problems. They will make decisions based on the mathematical solutions to the problem so that they will feel empowered by mathematics. In the course of solving problems, students will also evaluate their solutions, be critical about what they read and use technologies to carry out computations that may be too tedious to do manually.

No.	Topic/Sub-topics	Content
REAL-WORLD CONTEXTS		
R1	**Problems derived from real-world contexts**	• Real-world contexts such as * profit and loss (exclude use of the terms 'percentage profit' and 'percentage loss') * simple interest and compound interest[5] * household finance (earnings, expenditures, budgeting, etc.) * payment/ subscription rates (hire-purchase, utilities bills, etc.) * money exchange * time schedules (including 24-hour clock) and time zone variation * designs (models/structures, maps and plans, packaging, etc.) * everyday statistics (sport/ game statistics, household and market surveys, etc.) • tasks involving: * use of data from tables and charts * interpretation and use of graphs in practical situations * drawing graphs from given data * creating geometrical patterns and designs * interpretation and use of quantitative information

[5] For the GCE Exam, the compound interest formula to find the total amount will be provided.

Strand: Mathematical Processes

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. This includes *reasoning, communication and connections*, *applications and modelling*, and *thinking skills and heuristics* that are important in mathematics and beyond.

In the N(T) course, students learn to reason inductively and deductively, justifying and explaining their solutions. They are expected to write clear logical steps in presenting their solutions. They will continue to make connections among mathematical ideas, and between mathematics and the real world. They should be able to read critically, assessing the validity of arguments that are supported by mathematics in media.

Greater attention will be given to applications and some elements of modelling as students become more mature and aware of their immediate environment and phenomenon. Students build on their problem solving skills, use of heuristics, as well as the habit of checking on the reasonableness of answers. They work on real-world problems individually or in groups. They formulate a simple mathematical model to represent a real-world problem, apply mathematics to solve the problem, and interpret the solution and make informed decisions. The process of mathematical modelling widens and deepens students' understandings of mathematics, and helps them develop important 21st century skills, including collaboration, creativity, communication and critical thinking.

The teaching of process skills should be deliberate and yet integrated with the learning of concepts and skills. Students should be exposed to problem solving approach such as the Polya's model and the steps in a modelling process in class. Teachers could "think aloud" to give attention to these processes and make them visible to students. Students should be given opportunity to work in groups and use ICT tools for modelling tasks. ICT tools empower students to work on problems which would otherwise require more advanced mathematics or computations that are too tedious and repetitive. Through practice, students will develop habits and strategies that will help them be better and more independent learners.

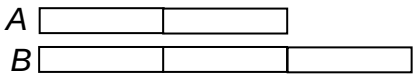
No.	Processes	Indicators
MATHEMATICAL PROCESSES		
MP1	**Reasoning, Communication and Connections**	• Use appropriate representations, mathematical language (including notations, symbols and conventions) and technology to present and communicate mathematical ideas • Reason inductively and deductively, including: ∗ Explaining or justifying/verifying a mathematical solution/statement ∗ Drawing logical conclusions ∗ Making inferences ∗ Writing mathematical arguments • Make connections within mathematics, and between mathematics and the real world
MP2	**Applications and Modelling**	• Apply mathematics concepts and skills to solve problems in a variety of contexts within or outside mathematics, including: ∗ Identifying the appropriate mathematical representations or standard models for a problem ∗ Using appropriate mathematical concepts, skills (including tools and algorithm) to solve a problem • Understand some elements of the mathematical modelling process, including: ∗ Formulating a simple mathematical model to represent a real world problem ∗ Applying mathematics to solve the problem ∗ Interpreting the mathematical solution and making informed decisions
MP3	**Thinking Skills and Heuristics**	• Use thinking skills such as: ∗ Classifying ∗ Comparing ∗ Sequencing ∗ Generalising ∗ Induction ∗ Deduction ∗ Analyzing (from whole to parts) ∗ Synthesizing (from parts to whole) • Use a problem-solving model such as Polya's model • Use heuristics such as: ∗ Drawing a diagram ∗ Tabulating ∗ Guess and check ∗ Working backwards ∗ Simplifying a problem ∗ Considering special cases

Content and Learning Experiences by Level

In this section, the content is listed by levels and learning experiences statements are included.

The learning experiences for the N(T)-Level Mathematics syllabus should provide opportunities for students to learn through:

- meaningful real-world contexts,
- practical hands-on experiences (including use of concrete manipulatives, etc)
- leveraging ICT

Content	Learning Experiences
SECONDARY ONE	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N1. Numbers and their operations	
1.1. negative numbers and primes (exclude prime factorisation) 1.2. integers and their four operations 1.3. four operations on fractions and decimals (including negative fractions and decimals) 1.4. calculations with calculator, including squares, cubes, square roots and cube roots 1.5. representation and ordering of numbers on the number line 1.6. use of <, >, ≤, ≥ 1.7. rounding off numbers to a required number of decimal places or significant figures 1.8. estimating the results of computation	(a) Discuss examples of negative numbers in the real world such as negative temperatures, places below sea-level, and time zone differences. (b) Compare big numbers up to billions and decimals in the context of money, mass and height. (c) Use algebra discs, or the AlgeDisc™ application in AlgeTools™, to make sense of addition, subtraction and multiplication involving negative integers and develop proficiency in the four operations of integers. (d) Discuss examples of fractions and decimals found in the real world such as recipes and supermarket shopping. (e) Explain the algebraic and geometric meanings of squares, cubes, square roots and cube roots, e.g. the cube root of 8 is the number whose cube is 8, and it is the edge length of a cube whose volume is 8 cubic units. (f) Relate <, >, ≤, ≥ with language such as 'less than', 'more than', 'less than or equal', 'more than or equal', 'at most' and 'at least'. (g) Estimate quantities (numbers and measures) to an appropriate degree of accuracy in a variety of contexts.
N2. Ratio and proportion	
2.1. comparison between two or more quantities by ratio 2.2. dividing a quantity in a given ratio 2.3. ratios involving fractions and decimals 2.4. equivalent ratios 2.5. writing a ratio in its simplest form	(a) Discuss and explain how ratios are used in real-world contexts such as recipes, maps, and golden ratio (human body/facial features). (b) Use the AlgeBar™ application in AlgeTools™ to express the ratio of 2 or 3 quantities in pictorial form. (c) Make connections between ratios and fractions and use appropriate mathematical language to describe the relationship, For example,  "The ratio of *A* to *B* is 2:3." is the same as "A is $\frac{2}{3}$ of B" ", or "*B* is $\frac{3}{2}$ of *A*".

Content	Learning Experiences
SECONDARY ONE	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N3. Percentage	
3.1. expressing percentage as a fraction or decimal 3.2. finding the whole given a percentage part 3.3. expressing one quantity as a percentage of another 3.4. comparing two quantities by percentage 3.5. percentages greater than 100% 3.6. finding one quantity given the percentage and the other quantity 3.7. increasing/decreasing a quantity by a given percentage 3.8. finding percentage increase/decrease	(a) Collect examples of percentages from newspapers and magazines and discuss the meaning of percentage in each example. (b) Examine bills and receipts, etc. to find examples of the uses of percentages such as discount, service charge, GST and other taxes and check the calculated values. (c) Discuss some misconceptions such as "If *A* is 10% more than *B*, then *B* is 10% less than *A*."
N5. Algebraic expressions and formulae	
5.1. using letters to represent numbers 5.2. interpreting notations: • ab as $a \times b$ • $\frac{a}{b}$ as $a \div b$ or $a \times \frac{1}{b}$ • a^2 as $a \times a$, a^3 as $a \times a \times a$, a^2b as $a \times a \times b$, . . . • $3y$ as $y + y + y$ or $3 \times y$ • $3(x+y)$ as $3 \times (x+y)$ • $\frac{3+y}{5}$ as $(3 + y) \div 5$ or $\frac{1}{5} \times (3 + y)$ 5.3. evaluation of algebraic expressions and formulae 5.4. translation of simple real-world situations into algebraic expressions 5.5. recognising and representing number sequences (include finding an algebraic expression for the *n*th term for simple cases such as $n+3, 2n+1$)	(a) Use a spreadsheet such as Microsoft Excel to * explore the concept of variables and evaluate algebraic expressions. * compare and examine the differences between pairs of expressions such as $2n$ and $2+n$, n^2 and $2n$, $2n^2$ and $(2n)^2$. (b) Use real-world examples to illustrate the meanings of simple algebraic expressions such as $3x+5$ and $\frac{y-10}{5}$. (c) Use algebra discs, or the AlgeDisc™ application in AlgeTools™, to make sense of and interpret linear expressions with integral coefficients such as $4x-3y$ and $-3(x-2)$. (d) Use the AlgeBar™ application in AlgeTools™ to formulate linear expressions (with integral coefficients) with pictorial representations. (e) Explore number sequences including those from real-world situations and write algebraic expressions to represent the sequences.

Content	Learning Experiences
SECONDARY ONE	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G1. Angles, triangles and quadrilaterals	
1.1. right, acute, obtuse and reflex angles 1.2. vertically opposite angles, angles on a straight line and angles at a point 1.3. angles formed by two parallel lines and a transversal: corresponding angles, alternate angles, interior angles	(a) Look for and discuss examples of different types of angles that can be found in their environment so that students would have a better understanding of the connection of geometry to the real world. (b) Use GSP or other dynamic geometry software to discover the relationships of the angles formed by two parallel lines and a transversal.
G2. Symmetry	
2.1. line and rotational symmetry of plane figures 2.2. lines of symmetry 2.3. order of rotational symmetry	(a) Discuss examples of everyday objects and shapes with line and rotational symmetry, identify the lines of symmetry and determine the order of rotational symmetry. (b) Explore and create symmetric figures and patterns, including with the use of ICT. (c) Use the ideas of reflection and mirror images to explain symmetric figures.
G4. Mensuration	
4.1. area of triangle as $\frac{1}{2}$ × base × height 4.2. area and circumference of circle 4.3. area of parallelogram and trapezium 4.4. problems involving perimeter and area of composite plane figures 4.5. visualising and sketching cube and cuboid (including use of nets to visualise the surface area of these solids) 4.6. volume and surface area of cube and cuboid 4.7. conversion between cm^2 and m^2, and between cm^3 and m^3 4.8. problems involving volume and surface area of composite solids	(a) Discuss real-world examples of triangles, circles, parallelograms, trapeziums, cubes and cuboids. (b) Use GSP or other dynamic geometry software to explore the properties of triangles, parallelograms, trapeziums and circles. (c) Relate the area of a triangle / parallelogram to that of a rectangle, and the area of a trapezium to that of a parallelogram. (d) Identify the height corresponding to any given side of a triangle or quadrilateral that is taken as the base. (e) Visualise and draw the nets of cubes and cuboids for the calculation of surface area.

Content	Learning Experiences
SECONDARY ONE	
STATISTICS AND PROBABILITY	**Students should have opportunities to:**
S1. Data Analysis	
1.1. analysis and interpretation of: • tables • bar graphs • pictograms • line graphs • pie charts 1.2. purposes and uses, advantages and disadvantages of the different forms of statistical representations	(a) Carry out a statistical project which involves data collection, representation and interpretation, involving the use of a spreadsheet such as Microsoft Excel to tabulate and represent data, e.g. conduct a survey of schoolmates on their favourite food and drinks for breakfast. (b) Explore different data representations used in the media (newspaper/internet) and comment on their appropriateness. (c) Explain how some data representations are misleading and can lead to erroneous interpretations. (d) Compare two sets of data using appropriate statistical diagrams such as line graphs on the same axes.
REAL-WORLD CONTEXTS	**Students should have opportunities to:**
R1. Problems derived from real-world	
1.1. Real-world contexts such as * profit and loss (exclude use of the terms 'percentage profit' and 'percentage loss') * household finance (earnings, expenditures, budgeting, etc.) * time schedules (including 24-hour clock) and time zone variation * designs (models/structures, packaging, etc.) * everyday statistics (sport/ game statistics, household and market surveys, etc.) 1.2. tasks involving: * use of data from tables and charts * interpretation and use of graphs in practical situations * drawing graphs from given data * creating geometrical patterns and designs * interpretation and use of quantitative information	(a) Examine and make sense of data in a variety of contexts (including real data presented in graphs and tables). (b) Work on tasks that incorporate some or all of the elements of the mathematical modelling process.

Content	Learning Experiences
SECONDARY TWO	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
2.6. direct and inverse proportion	(a) Examine real-life examples of inverse proportion, e.g. more workers would require less time to complete a job.
N4. Rate and speed	
4.1. rates and average rates (including the concepts of speed and average speed) 4.2. conversion of units (e.g. km/h to m/s)	(a) Discuss the concepts of rate and average rate using everyday activities such as typing, reading, pulse rate, and money exchange rates. (b) Find out and compare the speeds of bicycles, cars, trains and aeroplanes and their respective units to have a sense of their magnitudes. (c) Use a spreadsheet such as Microsoft Excel to compare the effects of simple interest and compound interest.
N5. Algebraic expressions and formulae	
5.6. addition and subtraction of linear expressions 5.7. simplification of linear expressions, e.g. • $-2(3x-5)+4x$ • $\frac{2x}{3}-\frac{3(x-5)}{2}$	(a) Use the AlgeDisc™ application in AlgeTools™ to construct linear expressions with integral coefficients, and simplify the expressions by collecting like terms and removing brackets.
N6. Functions and graphs	
6.1. Cartesian coordinates in two dimensions 6.2. graph of a set of ordered pairs as a representation of a relationship between two variables 6.3. linear functions $y=ax+b$ 6.4. graphs of linear functions 6.5. the gradient of a linear graph as the ratio of the vertical change to the horizontal change (positive and negative gradients)	(a) Play games, such as the Battleship Game, that involve the use of 2D Cartesian coordinates to specify points. (b) Examine real-world examples of linear functions and the concepts of constant increase and decrease. (c) Develop an understanding of function as a relationship between an independent variable (input) and a dependent variable (output), e.g. cost of water in a utility bill is a function of the number of units of water used. (d) Use a spreadsheet such as Microsoft Excel to produce a table of input and output for a given function describing the relationship in a real-life context, e.g. phone bill = basic subscription charge + utilisation charge. (e) Use a spreadsheet or graphing software to study how the graph of $y=ax+b$ changes when either a or b varies.

Content	Learning Experiences
SECONDARY TWO	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N7. Solutions of equations	
7.1. solving linear equations in one variable 7.2. formulating a linear equation in one variable to solve problems	(a) Build equations to represent real-life situations, e.g. the "mass of 5 plates and a 3-kg jug is 6 kg" is represented by $5x+3=6$, where x is the mass of one plate. (b) Use the virtual balance in AlgeTools™ to explore the concepts of equations, and to construct, simplify and solve linear equations with integral coefficients.
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G1. Angles, triangles and quadrilaterals	
1.4. properties of triangles and special quadrilaterals 1.5. properties of perpendicular bisectors of line segments and angle bisectors 1.6. construction of simple geometrical figures from given data (including perpendicular bisectors and angle bisectors) using compasses, ruler, set squares and protractor where appropriate	(a) Discuss examples of triangles and special quadrilaterals in the real world such as geometric designs and patterns. (b) Use GSP or other dynamic software to construct and study the properties of the perpendicular bisector of a line segment and the bisector of an angle. (c) Construct triangles given specific measurements of angles and sides (e.g. 2 sides and 1 angle) using a variety of tools including ICT.
G2. Congruence and similarity	
2.4. congruent and similar figures 2.5. properties of similar triangles and quadrilaterals: • corresponding angles are equal • corresponding sides are proportional	(a) Examine whether two figures are congruent, by checking if one figure can be mapped onto the other figure under translation, rotation and reflection (b) Use GSP or other dynamic software to draw, make measurements (of lengths, angles and areas) and explore the effects of translation, rotation, reflection and enlargement on the shape and size of a figure. (c) Identify and suggest applications of congruence and similarity in real-world contexts, e.g. photocopying, tessellation patterns, etc.

Content	Learning Experiences
SECONDARY TWO	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G3. Pythagoras' theorem	
3.1. use of Pythagoras' theorem 3.2. determining whether a triangle is right-angled given the lengths of three sides	(a) Either (i) use strings of 12 units (e.g. 1 unit = 10 cm) to form a right-angled triangle with sides of whole unit lengths (e.g. 3 units, 4 units and 5 units) and find out if there is a relationship involving the three sides; or (ii) use cut-out pieces of two squares with sides 3 units and 4 units respectively to form a square of sides 5 units. (b) Use drawings or GSP (or dynamic geometry software) to explore the validity/ invalidity of the theorem on different triangles and hence its use in showing if a triangle is right-angled.
G4. Mensuration	
4.9. visualising and sketching prism and cylinder (including use of nets to visualise the surface area of these solids) 4.10. volume and surface area of prism and cylinder 4.11. conversion between cm^2 and m^2, and between cm^3 and m^3	(a) Discuss examples of prisms and cylinders in the real world, e.g. containers and packaging. (b) Visualise and draw sketches of 3D shapes from different views (top, front and side views).
STATISTICS AND PROBABILITY	**Students should have opportunities to:**
S1. Data analysis	
1.3. analysis and interpretation of: • dot diagrams • histograms with equal class intervals 1.4. purposes and uses, advantages and disadvantages of the different forms of statistical representations 1.5. purposes and uses of mean, mode and median 1.6. calculation of the mean, mode and median for a set of ungrouped data	(a) Collect data and represent the data by a dot diagram or histogram. (b) Use a spreadsheet such as Microsoft Excel to show how the mean, mode and median are affected by changing data values. (c) Explain how some data representations are misleading and can lead to erroneous interpretations.

Content	Learning Experiences
SECONDARY TWO	
STATISTICS AND PROBABILITY	**Students should have opportunities to:**
S2. Probability	
2.1. probability as a measure of chance 2.2. probability of single events (including listing all the possible outcomes in a simple chance situation to calculate the probability)	(a) Discuss the use of probability, including predicting the outcome of matches, likelihood of an event, chance of getting a particular outcome, etc. (b) Conduct simple experiments using a dice, a spinner, or a coin, and explore the connections between empirical probability and theoretical probability, e.g. "Will there be 10 heads in 20 tosses of a fair coin?" (c) Discover the risk/chance-taking of gambling using examples such as the low chance of striking lottery and its disproportionate payout of the winnings.
REAL-WORLD CONTEXTS	**Students should have opportunities to:**
R1. Problems derived from real-world	
1.3. Real-world contexts such as * simple interest and compound interest[6] * payment/ subscription rates (hire-purchase, utilities bills, etc.) * money exchange * designs (models/structures, packaging, etc.) * everyday statistics (sport/ game statistics, household and market surveys, etc.) 1.4. tasks involving: * use of data from tables and charts * interpretation and use of graphs in practical situations * drawing graphs from given data * creating geometrical patterns and designs * interpretation and use of quantitative information	(a) Examine and make sense of data in a variety of contexts (including real data presented in graphs and tables). (b) Work on tasks that incorporate some or all of the elements of the mathematical modelling process.

[6] For the GCE Exam, the compound interest formula to find the total amount will be provided.

Content	Learning Experiences
SECONDARY THREE/FOUR	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N1. Numbers and their four operations	
1.9. use of index notation for integer powers 1.10. use of standard form $A \times 10^n$, where n is an integer, and $1 \leq A < 10$	(a) Discuss examples of millions (10^6), billions (10^9) and trillions (10^{12}) in real-world contexts, e.g. world population in billions, money in trillions.
N2. Ratio and proportion	
2.7. map scales (distance and area)	(a) Use local maps with map scales to find an actual distance and area. (b) Create maps of local places (e.g. school canteen) with appropriate map scales.
N4. Algebraic expressions and formulae	
5.8. expansion of the product of two linear expressions 5.9. multiplication and division of simple algebraic fractions, e.g. • $\left(\frac{3a}{4b^2}\right)\left(\frac{5ab}{3}\right)$ • $\frac{3a}{4} \div \frac{9a^2}{10}$ 5.10. changing the subject of a simple formula 5.11. finding the value of an unknown quantity in a given formula 5.12. factorisation of linear expressions of the form $ax + kay$ 5.13. factorisation of quadratic expressions of the form $x^2 + px + q$ Exclude: use of * $(a \pm b)^2 = a^2 \pm 2ab + b^2$ * $a^2 - b^2 = (a+b)(a-b)$ • addition and subtraction of algebraic fractions such as $\frac{1}{x} + \frac{1}{x-1}$	(a) Discuss real-world examples for expansion of the product of two linear expressions, such as the area of a rectangular field of side $(x-1)$ and $(x+4)$. (b) Use AlgeDisc™ to visualise and build conceptual understanding of the expansion process. (c) Relate factorisation as the reverse of expansion, including appropriate use of AlgeDisc™.

Content	Learning Experiences
SECONDARY THREE/FOUR	
NUMBER AND ALGEBRA	**Students should have opportunities to:**
N5. Functions and graphs	
6.6. graphs of linear equations in two variables ($ax+by=c$) 6.7. quadratic functions $y=ax^2+bx+c$ 6.8. graphs of quadratic functions and their properties • positive or negative coefficient of x^2 • maximum and minimum points • symmetry	(a) Use ICT (e.g. Graphmatica) to draw the graph of $ax+by=c$ (a straight line) and check that the coordinates of a point on the straight line satisfies the equation, and draw and describe the lines $x=a$ and $y=b$, and their gradients. (b) Examine real-world examples of the graphs of quadratic functions, e.g. the path of a basketball shot. (c) Use a spreadsheet or graphing software to study how the shape of the graph of $y=ax^2+bx+c$ changes when a, b or c varies.
N7. Solutions of equations	
7.3. solving simple fractional equations that can be reduced to linear equations, e.g. • $\frac{x}{3}+\frac{x-2}{4}=3$ • $\frac{3}{x-2}=6$ 7.4. solving simultaneous linear equations in two variables by • substitution and elimination methods • graphical method 7.5. solving quadratic equations in one variable by use of formula[7] 7.6. formulating a quadratic equation in one variable, a pair of linear equations in two variables to solve problems	(a) Translate a word problem into a pair of linear equations in two variables, and discuss how the equations can be solved. (b) Use ICT (e.g. Graphmatica) to draw the graphs of a pair of simultaneous linear equations, and explain why the solution is the point of intersection of the two straight lines.

[7] For the GCE Exam, the formula for solving quadratic equations will be provided.

Content	Learning Experiences
SECONDARY THREE/FOUR	
GEOMETRY AND MEASUREMENT	**Students should have opportunities to:**
G3. Trigonometry	
3.3. use of trigonometric ratios (sine, cosine and tangent) of acute angles to calculate unknown sides and angles in right-angled triangles (including problems involving angles of elevation and depression)	(a) Discuss the use of trigonometric ratios in real life, e.g. finding the height of a tree (or a building) by measuring the angle of elevation with a clinometer.
G4. Mensuration	
4.12. visualising and sketching pyramid, cone and sphere (including use of nets to visualise the surface area of these solids, where applicable) 4.13. volume and surface area of pyramid, cone and sphere[8] 4.14. conversion between cm^2 and m^2, and between cm^3 and m^3 4.15. arc length and sector area as fractions of the circumference and area of a circle	(a) Discuss real-world objects with the shapes of pyramids, cones and spheres, e.g. the pyramids in Egypt, ice-cream cones and soccer balls. (b) Use geometrical models or real-life objects to visualise and sketch pyramids, cones and spheres. (c) Make connections between the volume of a pyramid/cone and that of a prism/cylinder of the same height and base.
STATISTICS AND PROBABILITY	**Students should have opportunities to:**
S1. Data analysis	
1.7. percentiles, quartiles, range and interquartile range 1.8. analysis and interpretation of cumulative frequency diagrams	(a) Use a spreadsheet such as Microsoft Excel to construct a cumulative frequency diagram, and use it to estimate quartiles and percentiles.

[8] For the GCE Exam, the following formula will be provided: curved surface area of a cone, surface area of a sphere, volume of pyramid, cone and sphere.

Content	Learning Experiences
SECONDARY THREE/FOUR	
REAL-WORLD CONTEXTS	**Students should have opportunities to:**
R1. Problems derived from real-world	
1.5. Real-world contexts such as * designs (models/structures, maps and plans, packaging, etc.) * everyday statistics (sport/ game statistics, household and market surveys, etc.) 1.6. tasks involving: * use of data from tables and charts * interpretation and use of graphs in practical situations * drawing graphs from given data * creating geometrical patterns and designs * interpretation and use of quantitative information	(a) Examine and make sense of data in a variety of contexts (including real data presented in graphs and tables). (b) Work on tasks that incorporate some or all of the elements of the mathematical modelling process.