MATHEMATICS SYLLABUS

Pre-University

Higher 2

Syllabus 9758

Implementation starting with
2020 Pre-University One Cohort

Ministry of Education
SINGAPORE

Contents

SECTION 1: INTRODUCTION

Nature of Mathematics
Importance of Learning Mathematics
Mathematics at the A-Level
Mathematics Curriculum Framework
Mathematics and 21CC

1. Introduction

Nature of Mathematics

Mathematics can be described as a study of the *properties*, *relationships*, *operations*, *algorithms*, and *applications* of numbers and spaces at the very basic levels, and of abstract objects and concepts at the more advanced levels. Mathematical objects and concepts, and related knowledge and methods, are products of insight, logical reasoning and creative thinking, and are often inspired by problems that seek solutions. *Abstractions* are what make mathematics a powerful tool for solving problems. Mathematics provides within itself a language for *representing* and *communicating* the ideas and results of the discipline.

Importance of Learning Mathematics

Mathematics contributes to the developments and understanding in many disciplines and provides the foundation for many of today's innovations and tomorrow's solutions. It is used extensively to model and understand real-world phenomena (e.g. consumer preferences, population growth, and disease outbreak), create lifestyle and engineering products (e.g. animated films, mobile games, and autonomous vehicles), improve productivity, decision-making and security (e.g. business analytics, academic research and market survey, encryption, and recognition technologies).

In Singapore, mathematics education plays an important role in equipping every citizen with the necessary knowledge and skills and the capacities to think logically, critically and analytically to participate and strive in the future economy and society. In particular, for future engineers and scientists who are pushing the frontier of technologies, a strong foundation in mathematics is necessary as many of the Smart Nation initiatives that will impact the quality of lives in the future will depend heavily on computational power and mathematical insights.

Mathematics at the A-Level

There are four syllabuses to cater to the different needs, interests, and abilitites of students:
- H1 Mathematics;
- H2 Mathematics;
- H2 Further Mathematics; and
- H3 Mathematics.

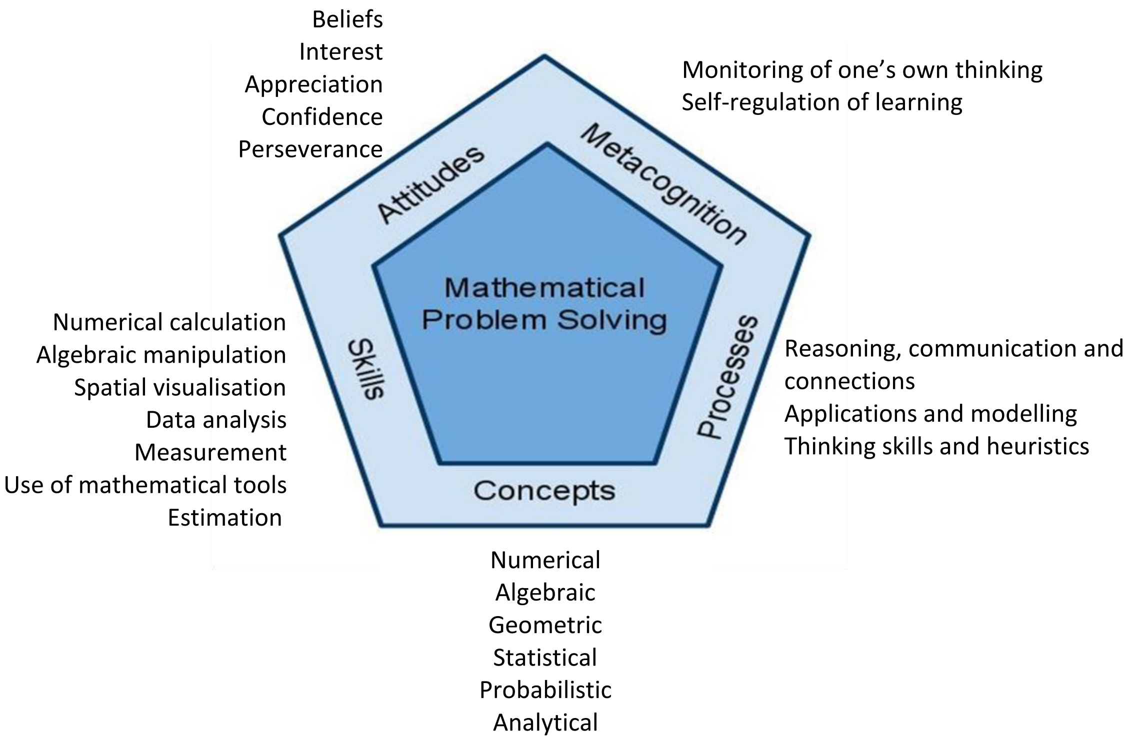
H2 Mathematics is designed to prepare students for a range of university courses, including mathematics, sciences and related courses, where a good foundation in mathematics is required. It develops mathematical thinking and reasoning skills that are essential for further learning of mathematics. Through the applications of mathematics, students also develop an appreciation of mathematics and its connections to other disciplines and to the real world.

Assumed knowledge: O-Level Additional Mathematics.

Learning mathematics at the A-Level provides students, regardless of the intended course of study at the university, with a useful set of tools and problem solving skills. It also exposes students to a way of thinking that complements other ways of thinking developed through the other disciplines.

Mathematics Curriculum Framework

- *Mathematical Problem Solving*



The central focus of the mathematics curriculum is the development of mathematical problem solving competency. Supporting this focus are five inter-related components – concepts, skills, processes, metacognition and attitudes. The framework sets the direction for and provides guidance in the teaching, learning, and assessment of mathematics.

- *Concepts*

Mathematical concepts can be broadly grouped into *numerical*, *algebraic*, *geometric*, *statistical*, *probabilistic*, and *analytical* concepts. These content categories are connected and interdependent. At different stages of learning and in different syllabuses, the breadth and depth of the content vary.

- *Skills*

Mathematical skills refer to *numerical calculation*, *algebraic manipulation*, *spatial visualisation*, *data analysis*, *measurement*, *use of mathematical tools*, and *estimation*. The skills are specific to mathematics and are important in the learning and application of mathematics. In today's classroom, these skills also include the abilities to use spreadsheets and other software to learn and do mathematics.

- *Processes*

Mathematical processes refer to the process skills involved in the process of acquiring and applying mathematical knowledge. These include *reasoning, communication and connections*, *applications and modelling*, and *thinking skills and heuristics* that are important in mathematics.

Reasoning, communication and connections:

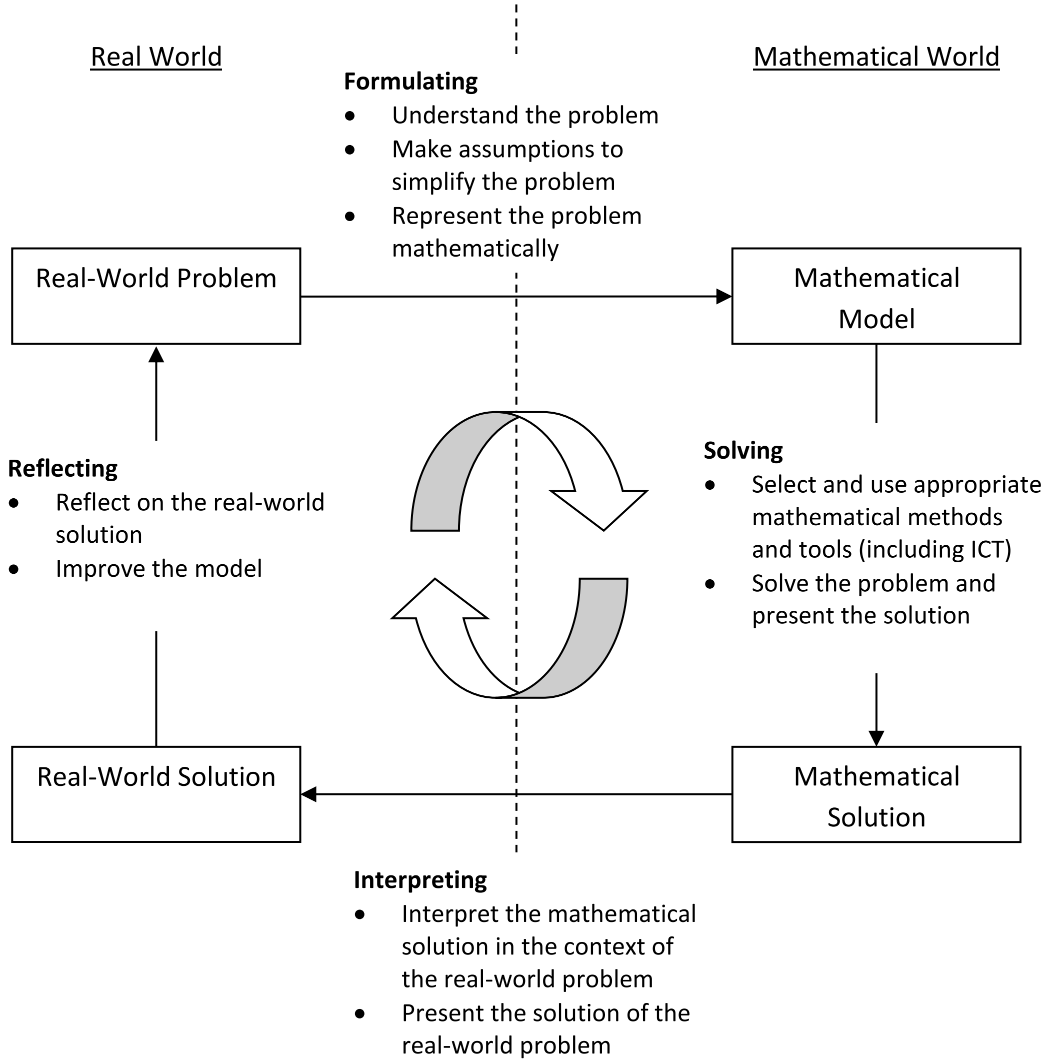
- Mathematical reasoning refers to the ability to analyse mathematical situations and construct logical arguments.
- Communication refers to the ability to use mathematical language to express mathematical ideas and arguments precisely, concisely and logically.
- Connections refer to the ability to see and make linkages among mathematical ideas, between mathematics and other subjects, and between mathematics and the real world.

Applications and modelling allow students to connect mathematics to the real world, enhance understanding of key mathematical concepts and methods, as well as develop mathematical competencies. Mathematical modelling is the process of formulating and improving a mathematical model[1] to represent and solve real-world problems. Through mathematical modelling, students learn to deal with complexity and ambiguity by simplifying and making reasonable assumptions, select and apply appropriate mathematical concepts and skills that are relevant to the problems, and interpret and evaluate the solutions in the context of the real-world problem. [The mathematical modelling process is shown in the diagram on the following page.]

Thinking skills and heuristics are essential for mathematical problem solving. Thinking skills refers to the ability to classify, compare, analyse, identify patterns and relationships, generalise, deduce and visualise. Heuristics are general strategies that students can use to solve non-routine problems. These include using a representation (e.g. drawing a diagram, tabulating), making a guess (e.g. trial and error/ guess and check, making a supposition), walking through the process (e.g. working backwards) and changing the problem (e.g. simplifying the problem, considering special cases).

[1] A mathematical model is a mathematical representation or idealisation of a real-world situation. It can be as complicated as a system of equations or as simple as a geometrical figure. As the word "model" suggests, it shares characteristics of the real-world situation that it seeks to represent.

Mathematical Modelling Process



- *Metacognition*

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring of one's own thinking, and self-regulation of learning.

- *Attitudes*

Attitudes refer to the affective aspects of mathematics learning such as:

- beliefs about mathematics and its usefulness;
- interest and enjoyment in learning mathematics;
- appreciation of the beauty and power of mathematics;
- confidence in using mathematics; and
- perseverance in solving a problem.

Mathematics and 21CC

Learning of mathematics creates opportunities for students to develop key competencies that are important in the 21st century. As an overarching approach, the A-Level mathematics curriculum supports the development of 21st century competencies (21CC) in the following ways:

1. The content are relevant to the needs of the 21st century. They provide the foundation for learning many of the advanced applications of mathematics that are relevant to today's world.

2. The pedagogies create opportunities for students to think critically, reason logically and communicate effectively, working individually as well as in groups, using ICT tools where appropriate in learning and doing mathematics.

3. The problem contexts raise students' awareness of local and global issues around them. For example, problems set around population issues and health issues can help students understand the challenges faced by Singapore and those around the world.

SECTION 2:
H2 MATHEMATICS SYLLABUS

2. H2 MATHEMATICS SYLLABUS (FROM 2020)

Preamble

Mathematics is a basic and important discipline that contributes to the developments and understandings of the sciences and other disciplines. It is used by scientists, engineers, business analysts and psychologists, etc. to model, understand and solve problems in their respective fields. A good foundation in mathematics and the ability to reason mathematically are therefore essential for students to be successful in their pursuit of various disciplines.

H2 Mathematics is designed to prepare students for a range of university courses, such as mathematics, sciences, engineering and related courses, where a good foundation in mathematics is required. It develops mathematical thinking and reasoning skills that are essential for further learning of mathematics. Through the applications of mathematics, students also develop an appreciation of mathematics and its connections to other disciplines and to the real world.

Syllabus Aims

The aims of H2 Mathematics are to enable students to:

(a) acquire mathematical concepts and skills to prepare for their tertiary studies in mathematics, sciences, engineering and other related disciplines;

(b) develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving;

(c) connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines; and

(d) experience and appreciate the nature and beauty of mathematics and its value in life and other disciplines.

Content Strands

There are 6 content strands in H2 Mathematics, namely, *Functions and Graphs*, *Sequences and Series*, *Vectors*, *Introduction to Complex Numbers*, *Calculus,* and *Probability and Statistics*.

a) <u>Functions and Graphs</u> provides a more abstract treatment of functions and their properties and studies the characteristics of a wider class of graphs including graphs defined parametrically, as well as transformation of graphs, techniques for solving equations, inequalities and system of equations.

b) Sequences and Series provides a useful tool for describing changes in discrete models and includes special series and sequences such as arithmetic and geometric progressions, and the concepts of convergence and infinity.

c) Vectors provides a useful tool for physical sciences as well as a means to describe and work with objects such as points, lines and planes in two- and three-dimensional spaces.

d) Complex Numbers provides an introduction to complex numbers as an extension of the number system and includes complex roots of polynomial equations, the four operations and the representation of complex numbers in exponential or polar forms.

e) Calculus provides useful tools for analysing and modelling change and behaviour and includes extension of differentiation and integration from Additional Mathematics, with additional techniques and applications such as power series and differential equations.

f) Probability and Statistics provides the foundation for modelling chance phenomena and making inferences with data and includes an introduction to counting techniques, computation of probability, general and specific discrete distribution models, normal distributions, sampling and hypothesis testing as well as correlation and regression.

There are many connections that can be made between the topics within each strand and across strands, even though the syllabus content are organised in strands. These connections should be emphasised as part of teaching and learning, to enable students to integrate the concepts and skills in a coherent manner to solve problems.

Knowledge of the content of O-Level Mathematics and part of Additional Mathematics is assumed in this syllabus.

Applications and Contexts

As H2 Mathematics is designed for students who intend to pursue a university course that is mathematics-related such as science and engineering, students should therefore be exposed to the applications of mathematics in science and engineering, so that they can appreciate the value and utility of mathematics in these likely courses of study.

The list illustrates the kinds of contexts that the mathematics learnt in the syllabus may be applied, and is by no means exhaustive.

Applications and contexts	Some possible topics involved
Kinematics and dynamics (e.g. free fall, projectile motion, collisions)	Functions; Calculus; Vectors

Optimisation problems (e.g. maximising strength, minimising surface area)	Inequalities; System of linear equations; Calculus
Electrical circuits	Complex numbers; Calculus
Population growth, radioactive decay, heating and cooling problems	Differential equations
Financial Maths (e.g. banking, insurance)	Sequences and series; Probability; Sampling distributions
Standardised testing	Normal distribution; Probability
Market research (e.g. consumer preferences, product claims)	Sampling distributions; Hypothesis testing; Correlation and regression
Clinical research (e.g. correlation studies)	Sampling distributions; Hypothesis testing; Correlation and regression

While students will be exposed to applications and contexts beyond mathematics, they are not expected to learn them in depth. Students should be able to use given information to formulate and solve the problems, applying the relevant concepts and skills and interpret the solution in the context of the problem.

Content

	Topics/ Sub-topics	Content
SECTION A: PURE MATHEMATICS		
1	**Functions and Graphs**	
1.1	Functions	Include: • concepts of function, domain and range • use of notations such as $\mathrm{f}(x)=x^2+5$, $\mathrm{f}: x \mapsto x^2+5$, $\mathrm{f}^{-1}(x)$, $\mathrm{fg}(x)$ and $\mathrm{f}^2(x)$ • finding inverse functions and composite functions • conditions for the existence of inverse functions and composite functions • domain restriction to obtain an inverse function • relationship between a function and its inverse Exclude the use of the relation $(\mathrm{fg})^{-1}=\mathrm{g}^{-1}\mathrm{f}^{-1}$, and restriction of domain to obtain a composite function.
1.2	Graphs and transformations	Include: • use of a graphing calculator to graph a given function • important characteristics of graphs such as symmetry, intersections with the axes, turning points and asymptotes of the following: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$; $\frac{y^2}{b^2}-\frac{x^2}{a^2}=1$ $y=\frac{ax+b}{cx+d}$ $y=\frac{ax^2+bx+c}{dx+e}$ • determining the equations of asymptotes, axes of symmetry, and restrictions on the possible values of x and/or y • effect of transformations on the graph of $y=\mathrm{f}(x)$ as represented by $y=a\mathrm{f}(x)$, $y=\mathrm{f}(x)+a$, $y=\mathrm{f}(x+a)$ and $y=\mathrm{f}(ax)$ and combinations of these transformations • relating the graphs of $y=\mathrm{f}^{-1}(x)$, $y=\lvert\mathrm{f}(x)\rvert$, $y=\mathrm{f}(\lvert x\rvert)$, and $y=\frac{1}{\mathrm{f}(x)}$ to the graph of $y=\mathrm{f}(x)$ • simple parametric equations and their graphs

	Topics/ Sub-topics	Content
1.3	Equations and inequalities	Include: • formulating an equation, a system of linear equations, or inequalities from a problem situation • solving an equation exactly or approximately using a graphing calculator • solving a system of linear equations using a graphing calculator • solving inequalities of the form $\frac{\text{f}(x)}{\text{g}(x)} > 0$ where $\text{f}(x)$ and $\text{g}(x)$ are linear expressions or quadratic expressions that are either factorisable or always positive • concept of $\lvert x \rvert$, and use of relations $\lvert x-a \rvert < b \Leftrightarrow a-b<x<a+b$ and $\lvert x-a \rvert > b \Leftrightarrow x<a-b$ or $x>a+b$, in the course of solving inequalities • solving inequalities by graphical methods
2	**Sequences and Series**	
2.1	Sequences and series	Include: • concepts of sequence and series for finite and infinite cases • sequence as function $y = \text{f}(n)$ where n is a positive integer • relationship between u_n (the nth term) and S_n (the sum to n terms) • sequence given by a formula for the nth term • use of $\sum$ notation • sum and difference of two series • summation of series by the method of differences • convergence of a series and the sum to infinity • formula for the nth term and the sum of a finite arithmetic series • formula for the nth term and the sum of a finite geometric series • condition for convergence of an infinite geometric series • formula for the sum to infinity of a convergent geometric series
3	**Vectors**	
3.1	Basic properties of vectors in two- and three-dimensions	Include: • addition and subtraction of vectors, multiplication of a vector by a scalar, and their geometrical interpretations • use of notations such as $\begin{pmatrix} x \\ y \end{pmatrix}$, $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$, $x\mathbf{i}+y\mathbf{j}$, $x\mathbf{i}+y\mathbf{j}+z\mathbf{k}$, $\overrightarrow{AB}$, $\mathbf{a}$ • position vectors, displacement vectors and direction vectors • magnitude of a vector

	Topics/ Sub-topics	Content
		• unit vectors • distance between two points • collinearity • use of the ratio theorem in geometrical applications
3.2	Scalar and vector products in vectors	Include: • concepts of scalar product and vector product of vectors and their properties • angle between two vectors • geometrical meanings of $\lvert \mathbf{a}\cdot\hat{\mathbf{n}} \rvert$ and $\lvert \mathbf{a}\times\hat{\mathbf{n}} \rvert$, where $\hat{\mathbf{n}}$ is a unit vector Exclude triple products $\mathbf{a}\cdot\mathbf{b}\times\mathbf{c}$ and $\mathbf{a}\times\mathbf{b}\times\mathbf{c}$.
3.3	Three-dimensional vector geometry	Include: • vector and cartesian equations of lines and planes • finding the foot of the perpendicular and distance from a point to a line or to a plane • finding the angle between two lines, between a line and a plane, or between two planes • relationships between (i) two lines (coplanar or skew) (ii) a line and a plane (iii) two planes Exclude: • finding the shortest distance between two skew lines • finding an equation for the common perpendicular to two skew lines
4	**Introduction to Complex Numbers**	
4.1	Complex numbers expressed in Cartesian form	Include: • extension of the number system from real numbers to complex numbers • complex roots of quadratic equations • conjugate of a complex number • four operations of complex numbers • equality of complex numbers • conjugate roots of a polynomial equation with real coefficients
4.2	Complex numbers expressed in polar form	Include: • representation of complex numbers in the Argand diagram • complex numbers expressed in the form $r(\cos\theta + \mathrm{i}\sin\theta)$, or $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \le \pi$ • calculation of modulus (r) and argument (θ) of a complex number • multiplication and division of two complex numbers expressed in polar form

	Topics/ Sub-topics	Content
5	**Calculus**	
5.1	Differentiation	Include: • graphical interpretation of (i) $\mathrm{f}'(x)>0$, $\mathrm{f}'(x)=0$ and $\mathrm{f}'(x)<0$ (ii) $\mathrm{f}''(x)>0$ and $\mathrm{f}''(x)<0$ • relating the graph of $y=\mathrm{f}'(x)$ to the graph of $y=\mathrm{f}(x)$ • differentiation of simple functions defined implicitly or parametrically • determining the nature of the stationary points (local maximum and minimum points and points of inflexion) analytically, in simple cases, using the first derivative test or the second derivative test • locating maximum and minimum points using a graphing calculator • finding the approximate value of a derivative at a given point using a graphing calculator • finding equations of tangents and normals to curves, including cases where the curve is defined implicitly or parametrically • local maxima and minima problems • connected rates of change problems Exclude finding non-stationary points of inflexion and finding second derivative of functions defined parametrically.
5.2	Maclaurin series	Include: • standard series expansion of $(1+x)^n$ for any rational n, $\mathrm{e}^x, \sin x, \cos x$ and $\ln(1+x)$ • derivation of the first few terms of the Maclaurin series by o repeated differentiation, e.g. $\sec x$ o repeated implicit differentiation, e.g. $y^3+y^2+y=x^2-2x$ o using standard series, e.g. $\mathrm{e}^x\cos 2x$, $\ln\left(\frac{1+x}{1-x}\right)$ • range of values of x for which a standard series converges • concept of "approximation" • small angle approximations: $\sin x \approx x$, $\cos x \approx 1-\frac{1}{2}x^2$, $\tan x \approx x$ Exclude derivation of the general term of the series.

	Topics/ Sub-topics	Content
5.3	Integration techniques	Include: • integration of $f'(x)[f(x)]^n$ (including $n=-1$), $f'(x)e^{f(x)}$ $\sin^2 x, \cos^2 x, \tan^2 x$, $\sin mx \cos nx, \cos mx \cos nx$ and $\sin mx \sin nx$ $\frac{1}{a^2+x^2}, \frac{1}{\sqrt{a^2-x^2}}, \frac{1}{a^2-x^2}$ and $\frac{1}{x^2-a^2}$ • integration by a given substitution • integration by parts
5.4	Definite integrals	Include: • concept of definite integral as a limit of sum • definite integral as the area under a curve • evaluation of definite integrals • finding the area of a region bounded by a curve and lines parallel to the coordinate axes, between a curve and a line, or between two curves • area below the x-axis • finding the area under a curve defined parametrically • finding the volume of revolution about the x- or y-axis • finding the approximate value of a definite integral using a graphing calculator Exclude finding the volume of revolution about the x-axis or y-axis where curve is defined parametrically.
5.5	Differential equations	Include: • solving for the general solutions and particular solutions of differential equations of the forms (i) $\frac{dy}{dx} = f(x)$ (ii) $\frac{dy}{dx} = f(y)$ (iii) $\frac{d^2y}{dx^2} = f(x)$ including those that can be reduced to (i) and (ii) by means of a given substitution • formulating a differential equation from a problem situation • interpreting a differential equation and its solution in terms of a problem situation

	Topics/ Sub-topics	Content
SECTION B: PROBABILITY AND STATISTICS		
6	**Probability and Statistics**	
6.1	Probability	Include: • addition and multiplication principles for counting • concepts of permutation (^nP_r) and combination (^nC_r) • arrangements of objects in a line or in a circle, including cases involving repetition and restriction • addition and multiplication of probabilities • mutually exclusive events and independent events • use of tables of outcomes, Venn diagrams, tree diagrams, and permutations and combinations techniques to calculate probabilities • calculation of conditional probabilities in simple cases • use of: $\mathrm{P}(A') = 1 - \mathrm{P}(A)$ $\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$ $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$
6.2	Discrete random variables	Include: • concept of discrete random variables, probability distributions, expectations and variances • concept of binomial distribution $\mathrm{B}(n, p)$ as an example of a discrete probability distribution and use of $\mathrm{B}(n, p)$ as a probability model, including conditions under which the binomial distribution is a suitable model • use of mean and variance of binomial distribution (without proof) Exclude finding cumulative distribution function of a discrete random variable.
6.3	Normal distribution	Include: • concept of a normal distribution as an example of a continuous probability model and its mean and variance; use of $\mathrm{N}(\mu, \sigma^2)$ as a probability model • standard normal distribution • finding the value of $\mathrm{P}(X < x_1)$ or a related probability, given the values of x_1, μ, σ • symmetry of the normal curve and its properties • finding a relationship between x_1, μ, σ given the value of $\mathrm{P}(X < x_1)$ or a related probability • solving problems involving the use of $\mathrm{E}(aX + b)$ and $\mathrm{Var}(aX + b)$ • solving problems involving the use of $\mathrm{E}(aX + bY)$ and $\mathrm{Var}(aX + bY)$, where X and Y are independent Exclude normal approximation to binomial distribution.

	Topics/ Sub-topics	Content
6.4	Sampling	Include: • concepts of population and simple random sample • concept of the sample mean $\overline{X}$ as a random variable with $\mathrm{E}(\overline{X})=\mu$ and $\mathrm{Var}(\overline{X})=\frac{\sigma^2}{n}$ • distribution of sample mean from a normal population • use of the Central Limit Theorem to treat sample mean as having normal distribution when the sample size is sufficiently large (e.g. $n \geq 30$) • calculation and use of unbiased estimates of the population mean and variance from a sample, including cases where the data are given in summarised form $\sum x$ and $\sum x^2$, or $\sum(x-a)$ and $\sum(x-a)^2$
6.5	Hypothesis testing	Include: • concepts of null hypothesis (H_0) and alternative hypotheses (H_1), test statistic, critical region, critical value, level of significance, and *p*-value • formulation of hypotheses and testing for a population mean based on: - a sample from a normal population of known variance - a large sample from any population • 1-tail and 2-tail tests • Interpretation of the results of a hypothesis test in the context of the problem Exclude the use of the term 'Type I error', concept of Type II error and testing the difference between two population means.

	Topics/ Sub-topics	Content
6.6	Correlation and Linear regression	Include: • use of scatter diagram to determine if there is a plausible linear relationship between the two variables • correlation coefficient as a measure of the fit of a linear model to the scatter diagram • finding and interpreting the product moment correlation coefficient (in particular, values close to -1, 0 and 1) • concepts of linear regression and method of least squares to find the equation of the regression line • concepts of interpolation and extrapolation • use of the appropriate regression line to make prediction or estimate a value in practical situations, including explaining how well the situation is modelled by the linear regression model • use of a square, reciprocal or logarithmic transformation to achieve linearity Exclude: • derivation of formulae • relationship $r^2 = b_1 b_2$, where b_1 and b_2 are regression coefficients • hypothesis tests

SECTION 3:
PEDAGOGY AND
FORMATIVE ASSESSMENT

Teaching Processes
Phases of Learning
Formative Assessment
Use of Technology

3. PEDAGOGY AND FORMATIVE ASSESSMENT

Teaching Processes

The Pedagogical Practices of The Singapore Teaching Practice (STP) outlines four Teaching Processes that make explicit what teachers reflect on and put into practice before, during and after their interaction with students in all learning contexts.

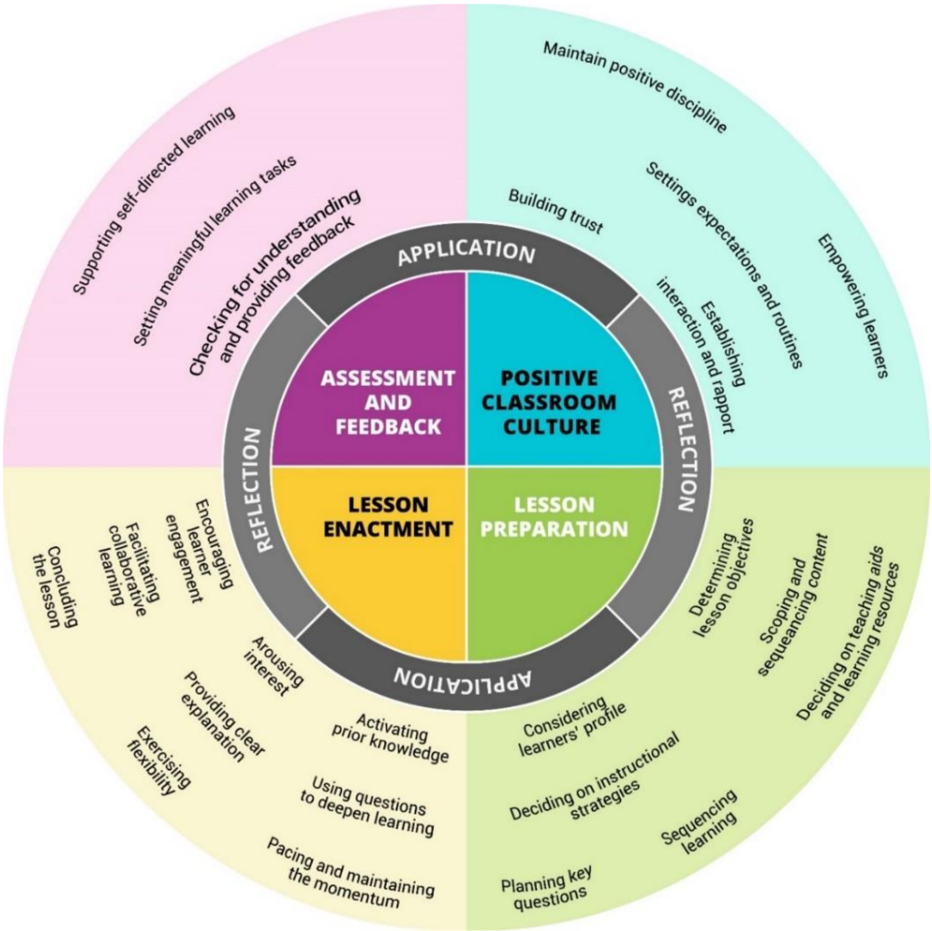
It is important to view the Pedagogical Practices of the STP in the context of the Singapore Curriculum Philosophy (SCP) and Knowledge Bases (KB), and also to understand how all three components work together to support effective teaching and learning.

Taking reference from the SCP, every student is valued as an individual, and they have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge, and skills. For learning to be effective, there is a need to adapt and match the teaching pace, approaches and assessment practices so that they are developmentally appropriate.

The 4 Teaching Processes are further expanded into Teaching Areas as follows:

Assessment and Feedback • Checking for Understanding and Providing Feedback • Supporting Self-Directed Learning • Setting Meaningful Assignments	Positive Classroom Culture • Establishing Interaction and Rapport • Maintaining Positive Discipline • Setting Expectations and Routines • Building Trust • Empowering Learners
Lesson Enactment • Activating Prior Knowledge • Arousing Interest • Encouraging Learner Engagement • Exercising Flexibility • Providing Clear Explanation • Pacing and Maintaining Momentum • Facilitating Collaborative Learning • Using Questions to Deepen Learning • Concluding the Lesson	Lesson Preparation • Determining Lesson Objectives • Considering Learners' Profile • Selecting and Sequencing Content • Planning Key Questions • Sequencing Learning • Deciding on Instructional Strategies • Deciding on Teaching Aids and Learning Resources

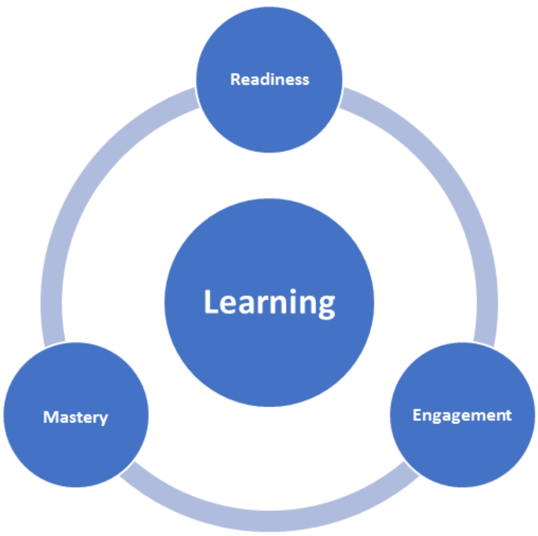
The Teaching Areas are not necessarily specific to a single Teaching Process. Depending on the context, some of the Teaching Areas could be considered in another Teaching Process. The Teaching Processes are undergirded by a constant cycle of application and reflection.



For more information on STP, go to https://www.moe.gov.sg/about/singapore-teaching-practice

Phases of Learning

The Teaching Areas in STP are evident in the effective planning and delivery of the three phases of learning - *readiness*, *engagement* and *mastery*.



Readiness Phase

Student readiness to learn is vital to learning success. Teachers have to consider the following:

- Learning environment
- Students' profile
- Students' prior and pre-requisite knowledge
- Motivating contexts

Engagement Phase

This is the main phase of learning where students engage with the new materials to be learnt (*Encouraging Learner Engagement*). As students have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge and skills, it is important to consider the pace of the learning and transitions (*Pacing and Maintaining Momentum*) using a repertoire of pedagogies.

Three pedagogical approaches form the spine that supports most of the mathematics instruction in the classroom. They are not mutually exclusive and could be used in different parts of a lesson or unit. Teachers make deliberate choices on the instructional strategies (Deciding on Instructional Strategies) based on learners' profiles and needs, and the nature of the concepts to be taught. The engagement phase can include one or more of the following:

- Activity-based Learning
- Inquiry-based Learning
- Direct Instruction

Regardless of the approach, it is important for teachers to plan ahead, anticipate students' responses, and adapt the lesson accordingly (Exercising Flexibility).

Mastery Phase

The mastery phase is the final phase of learning where students consolidate and extend their learning. To consolidate, teachers summarise and review key learning points at the end of a lesson and make connections with the subsequent lesson (Concluding the Lesson). The mastery phase can include one or more of the following:

- Motivated Practice
- Reflective Review
- Extended Learning

Formative Assessment

Assessment is an integral part of the teaching and learning. It can be formative or summative or both. Formative assessment or Assessment for Learning (AfL) is carried out during teaching and learning to gather evidence and information about students' learning.

The *purpose* of formative assessment is to help students improve their learning and be self-directed in their learning. In learning of mathematics, just as in other subjects, information about students' understanding of the content must be gathered *before*, *during* and *after* the lesson.

The outcomes of the mathematics curriculum go beyond just the recall of mathematical concepts and skills. Since mathematical problem solving is the focus of the mathematics curriculum, assessment should also focus on students' understanding and ability to apply what they know to solve problems. In addition, there should be emphasis on processes such as reasoning, communicating, and modelling.

The overarching objectives of assessment should focus on students':

- understanding of mathematical concepts (going beyond simple recall of facts);
- ability to reason, communicate, and make meaningful connections and integrate ideas across topics;
- ability to formulate, represent and solve problems within mathematics and to interpret mathematical solutions in the context of the problems; and
- ability to develop strategies to solve non-routine problems.

The process of assessment is embedded in the planning of the lessons. The embedding of assessment process may take the following forms:

- Class Activities
- Classroom Discourse
- Individual or Group Tasks

Assessment provides feedback for both students and teachers.

- Feedback from teachers to students informs students where they are in their learning and what they need to do to improve their learning.
- Feedback from students to teachers comes from their responses to the assessment tasks designed by teachers. They provide information to teachers on what they need to do to address learning gaps, how to modify the learning activities students engage in, and how they should improve their instruction.
- Feedback between students is important as well because peer-assessment is useful in promoting active learning. It provides an opportunity for students to learn from each other and also allows them to develop an understanding of what counts as quality work by critiquing their peers' work in relation to a particular learning outcome.

Use of Technology

Computational tools are essential in many branches of mathematics. They support the discovery of mathematical results and applications of mathematics. Mathematicians use computers to solve computationally challenging problems, explore new ideas, form conjectures and prove theorems. Many of the applications of mathematics rely on the availability of computing power to perform operations at high speed and on a large scale. Therefore, integrating technology into the learning of mathematics gives students a glimpse of the tools and practices of mathematicians.

Computational tools are also essential for the learning of mathematics. In particular, they support the understanding of concepts (e.g. simulation and digital manipulatives), their properties (e.g. geometrical properties) and relationships (e.g. algebraic form versus graphical form). More generally, they can be used to carry out investigation (e.g. dynamic geometry software, graphing tools and spreadsheets), communicate ideas (e.g. presentation tools) and collaborate with one another as part of the knowledge building process (e.g. discussion forum). Getting students who have experience with coding to implement some of the algorithms in mathematics (e.g. finding prime factors, multiplying two matrices) can potentially help these students develop a clearer understanding of the algorithms and the underlying mathematics concepts as well.

SECTION 4:
SUMMATIVE ASSESSMENT

Purpose and Assessment Objectives
National Examination (Syllabus 9758)

4. SUMMATIVE ASSESSMENT

Purpose and Assessment Objectives

The purpose of summative assessments, such as tests and examinations, is to measure the extent to which students have achieved the learning objectives of the syllabuses.

The assessment objectives reflect the emphases of the syllabus and describe what students should know and be able to do with the concepts and skills learned.

National Examination: H2 Mathematics (Syllabus 9758)

Important information on the national examination for H2 Mathematics is highlighted below. Full details are available on the SEAB website.

The examination will be based on the topics/content listed in Section 2. Knowledge of O-Level Mathematics and some of the content of O-Level Additional Mathematics are assumed.

The use of an approved graphing calculator will be expected.

ASSESSMENT OBJECTIVES (AO)
There are three levels of assessment objectives for the examination.

The assessment will test candidates' abilities to:

AO1 Understand and apply mathematical concepts and skills in a variety of problems, including those that may be set in unfamiliar contexts, or require integration of concepts and skills from more than one topic.

AO2 Formulate real-world problems mathematically, solve the mathematical problems, interpret and evaluate the mathematical solutions in the context of the problems.

AO3 Reason and communicate mathematically through making deductions and writing mathematical explanations, arguments and proofs.

Notwithstanding the presentation of the topics in the syllabus document, it is envisaged that some examination questions may integrate ideas from more than one topic, and that topics may be tested in the contexts of problem solving and application of mathematics. While problems may be set in context, no assumptions will be made about the knowledge of the context. All information will be self-contained within the problem.

SCHEME OF EXAMINATION PAPERS

For the examination in H2 Mathematics, there will be two 3-hour papers, each carrying 50% of the total mark, and each marked out of 100, as follows:

PAPER 1 (3 hours)

A paper consisting of 10 to 12 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

There will be at least two questions on application of Mathematics in real-world contexts, including those from sciences and engineering. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Candidates will be expected to answer all questions.

PAPER 2 (3 hours)

A paper consisting of two sections, Sections A and B.

Section A (Pure Mathematics – 40 marks) will consist of 4 to 5 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.

Section B (Probability and Statistics – 60 marks) will consist of 6 to 8 questions of different lengths and marks based on the Probability and Statistics section of the syllabus.

There will be at least two questions in Section B on application of Mathematics in real-world contexts, including those from sciences and engineering. Each question will carry at least 12 marks and may require concepts and skills from more than one topic.

Candidates will be expected to answer all questions.

The assumed knowledge for O-Level Additional Mathematics is stated below.

Content from O Level Additional Mathematics	
ALGEBRA	
A1	Equations and inequalities • conditions for a quadratic equation to have: (i) two real roots (ii) two equal roots (iii) no real roots • conditions for ax^2+bx+c to be always positive (or always negative) • solving simultaneous equations with at least one linear equation, by substitution
A2	Indices and surds • four operations on indices and surds • rationalising the denominator
A3	Polynomials and partial fractions • multiplication and division of polynomials • use of remainder and factor theorems • partial fractions with cases where the denominator is not more

Content from O Level Additional Mathematics	
	complicated than: - $(ax + b)(cx + d)$ - $(ax + b)(cx + d)^2$ - $(ax + b)(x^2 + c^2)$
A4	Power, Exponential, Logarithmic, and Modulus functions • power functions $y = ax^n$, where n is a simple rational number, and their graphs • functions a^x, e^x, $\log_a x$, $\ln x$ and their graphs • laws of logarithms • equivalence of $y = a^x$ and $x = \log_a y$ • change of base of logarithms • function $\|x\|$ and graph of $\|f(x)\|$, where $f(x)$ is linear, quadratic or trigonometric • solving simple equations involving exponential and logarithmic functions
GEOMETRY AND TRIGONOMETRY	
B5	Coordinate geometry in two dimensions • graphs of equations $y^2 = kx$ • coordinate geometry of the circle with the equation in the form $(x-a)^2 + (y-b)^2 = r^2$ or $x^2 + y^2 + 2gx + 2fy + c = 0$
B6	Trigonometric functions, identities and equations • six trigonometric functions, and principal values of the inverses of sine, cosine and tangent • trigonometric equations and identities (see List of Formulae) • expression of $a\cos\theta + b\sin\theta$ in the forms $R\sin(\theta \pm \alpha)$ and $R\cos(\theta \pm \alpha)$
CALCULUS	
C7	Differentiation and integration • derivative of $f(x)$ as the gradient of the tangent to the graph of $y = f(x)$ at a point • derivative as rate of change • derivatives of x^n for any rational n, $\sin x$, $\cos x$, $\tan x$, e^x and $\ln x$, together with constant multiples, sums and differences • derivatives of composite functions • derivatives of products and quotients of functions • increasing and decreasing functions • stationary points (maximum and minimum turning points and points of inflexion) • use of second derivative test to discriminate between maxima and minima • connected rates of change • maxima and minima problems • integration as the reverse of differentiation • integration of x^n for any rational n, e^x, $\sin x$, $\cos x$, $\sec^2 x$ and their constant multiples, sums and differences • integration of $(ax+b)^n$ for any rational n, $\sin(ax+b)$, $\cos(ax+b)$ and e^{ax+b}