# FURTHER MATHEMATICS

# SYLLABUS

## Pre-University

## Higher 2

## Syllabus 9649

Implementation starting with
2024 Pre-University One Cohort

Ministry of Education
SINGAPORE

# Contents

# SECTION 1: INTRODUCTION

Importance of Learning Mathematics
Mathematics at the A-Level
2024 A-Level Mathematics Curriculum

# 1. Introduction

**Importance of Learning Mathematics**

Mathematics contributes to the developments and understanding in many disciplines and provides the foundation for many of today's innovations and tomorrow's solutions. It is used extensively to model and understand real-world phenomena (e.g. consumer preferences, population growth, and disease outbreak), create lifestyle and engineering products (e.g. animated films, mobile games, and autonomous vehicles), improve productivity, decision-making and security (e.g. business analytics, academic research and market survey, encryption, and recognition technologies).

In Singapore, mathematics education plays an important role in equipping every citizen with the necessary knowledge and skills and the capacities to think logically, critically and analytically to participate and strive in the future economy and society. In particular, for future engineers and scientists who are pushing the frontier of technologies, a strong foundation in mathematics is necessary as many of the Smart Nation initiatives that will impact the quality of lives in the future will depend heavily on computational power and mathematical insights.

**Mathematics at the A-Level**

There are four syllabuses to cater to the different needs, interests, and abilities of students:

- H1 Mathematics;
- H2 Mathematics;
- H2 Further Mathematics; and
- H3 Mathematics.

**H2 Further Mathematics** is designed for students who are mathematically-inclined and who wish to further expand and deepen their knowledge of mathematics and its applications. Students will develop advanced mathematical thinking and reasoning skills and learn a wider range of mathematical methods and tools. This will give students who plan to study mathematics or mathematics-related university courses such as science and engineering a head start in the form of a stronger and richer foundation in mathematics.

**H2 Further Mathematics is offered with H2 Mathematics as a double mathematics course.**

Learning mathematics at the A-Level provides students, regardless of the intended course of study at the university, with a useful set of tools and problem solving skills. It also exposes students to a way of thinking that complements other ways of thinking developed through the other disciplines.

**2024 A-Level Mathematics Curriculum**

The 2024 A-Level Mathematics Curriculum is an update of the 2016 A-Level Mathematics Curriculum and incorporates the key shifts in the 2020 Secondary Mathematics Curriculum. All the syllabuses will continue to emphasise the development of mathematical processes and highlight applications of mathematics. In addition, the 2024 A-level Mathematics Curriculum will emphasise the following:

a) *Strengthening mathematical practices*: These are practices that enable students to seek problems and learn mathematics on their own, construct knowledge and communicate their ideas mathematically. More opportunities should be created for students to be engaged in such practices, which also support the development of 21st century competencies.

b) *Using computers as mathematical tools*: This goes beyond the use of computers for teaching and learning, but for doing mathematical work. It creates opportunities for students to develop computational thinking that is also encouraged at the secondary level. Learning objectives that explicitly mention the use of computers and software/apps as mathematical tools are included, where appropriate, but will not be examinable.

c) *Teaching towards big ideas*: This will strengthen students' appreciation and deepen their understanding of mathematics, and will encourage students to see beyond the topics, and also their connections. It is a continuation from their learning experiences at the secondary level, where teaching towards big ideas is emphasised. The themes and big ideas that are featured in the 2024 A-Level Mathematics Curriculum are described in Section 2.

# SECTION 2: MATHEMATICS CURRICULUM

Nature of Mathematics
Themes and Big Ideas
Mathematics Curriculum Framework
Mathematics and 21st Century Competencies

# 2. Mathematics Curriculum

### Nature of Mathematics

Mathematics can be described as a study of the *properties*, *relationships*, *operations*, *algorithms*, and *applications* of numbers and spaces at the very basic levels, and of abstract objects and concepts at the more advanced levels. Mathematical objects and concepts, and related knowledge and methods, are products of insight, logical reasoning and creative thinking, and are often inspired by problems that seek solutions. *Abstractions* are what make mathematics a powerful tool for solving problems. Mathematics provides within itself a language for *representing* and *communicating* the ideas and results of the discipline.

### Themes and Big Ideas

From the above description of the nature of mathematics, four recurring themes in the study of mathematics are derived: *Properties and Relationships*, *Operations and Algorithms*, *Representations and Communications*, and *Abstractions and Applications*.


1. *Properties and Relationships*: What are the properties of mathematical objects and how are they related?

   *Properties* of mathematical objects (e.g. numbers, lines, function, etc.) are either inherent in their definitions or derived through logical argument and rigorous proof. *Relationships* exist between mathematical objects. They include the equivalence of two expressions or statements, the connections between two functions, relationship between vector equations of lines and planes, and relationship between independent and dependent variables. Understanding properties and relationships enable us to gain deeper insights into the mathematical objects and use them to model and solve real-world problems.

2. *Operations and Algorithms*: What meaningful actions can we perform on the mathematical objects and how do we carry them out?

   *Operations* are meaningful actions performed on mathematical objects. They include algebraic manipulations, geometric transformations, operations on functions, and many more. *Algorithms* are generalised sequences of well-defined smaller steps to perform a mathematical operation or to solve a problem. An example is the root-finding algorithms to approximate the roots of an equation (e.g. using Newton-Raphson method) or solutions of first order differential equations (e.g. using Euler method). Understanding the meaning of these operations and algorithms and how to carry them out enable us to solve problems mathematically.

3. *Representations and Communications*: How can the mathematical objects and concepts be represented and communicated within and beyond the discipline?

   *Representations* are integral to the language of mathematics. They include symbols, notations, and diagrams such as graphs, geometrical figures, Venn diagrams and tree diagrams that are used to express mathematical concepts, properties and operations in a way that is precise and universally understood. *Communication* of mathematics is necessary for the understanding and dissemination of knowledge within the community of practitioners as well as general public. It includes clear presentation of proof in technical writing as well as choosing appropriate representations (e.g. stating null and alternative hypotheses, using a scatter diagram to represent relationship between two variables) to communicate mathematical ideas that can be understood by the masses.

4. *Abstractions and Applications*: How can the mathematical objects be further abstracted and where can they be applied?

   *Abstraction* is at the core of mathematical thinking. It involves the process of generalisation, extension and synthesis. Through algebra, we generalise arithmetic. Through complex numbers, we extend the number system. Through coordinate geometry, we synthesise the concepts across the algebra and geometry strands. The processes of abstraction make visible the structure and rich connections within mathematics and makes mathematics a powerful tool. *Application* of mathematics is made possible by abstractions. From simple counting to complex modelling, the abstract mathematical objects, properties, operations, relationships and representations can be used to model and study real-world phenomena.

Big ideas express ideas that are *central* to mathematics. They appear in different topics and strands. There is a *continuation* of the ideas across levels. They bring *coherence* and show *connections* across different topics, strands and levels. The big ideas in mathematics could be about one or more themes, that is, it could be about *properties and relationships* of mathematical objects and concepts and the *operations and algorithms* involving these objects and concepts, or it could be about *abstraction and applications* alone. Understanding the big ideas brings one closer to appreciating the nature of mathematics.

Eight clusters of big ideas are listed in 2024 A-Level Mathematics Curriculum. Each cluster of big ideas is represented by a label for ease of reference: **Functions**, **Diagrams**, **Models**, **Equivalence**, **Transformation**, **Limits**, **Vectors**, and **Extensions**. They relate to the four themes that cut across and connect concepts from the different content strands. Some big ideas extend across and connect more concepts than others, and some also extend from the big ideas in the 2020 Secondary Mathematics Curriculum[1]. The list of big ideas is not meant to be authoritative or comprehensive.

A brief description of the big ideas in the 2024 A-Level Mathematics Curriculum is given below.

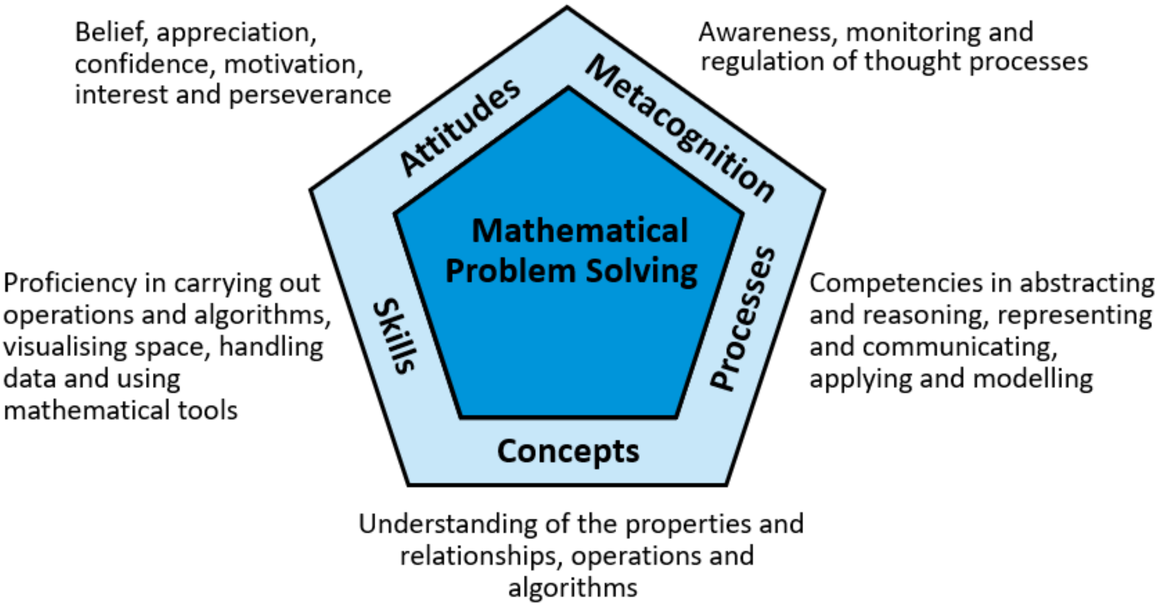
| | |
|---|---|
| **FUNCTIONS**<br>(Main Theme: Properties and Relationships)<br><br>Functions express the relationship between two sets of mathematical objects by a rule that maps the elements of one set to those in the other set. The rule of a function may be expressed verbally, algebraically, numerically (as a table) or graphically (as a graph). Functions may have inverse and can be combined. Many operations and algorithms in mathematics can be thought of as a function, with appropriate input, rule and output. The input and output need not be limited to real numbers. This conceptualisation of function is useful when thinking about how to implement an operation or algorithm as codes in computer. | **DIAGRAMS**<br>(Main Theme: Representations and Communications)<br><br>Real-world or mathematical objects can be represented succinctly and visually using mathematical diagrams. Diagrams serve to communicate properties of the objects, show the relationship between objects and facilitate problem solving. Understanding what different diagrams represent, their features and conventions, and how they are constructed helps us understand and communicate mathematical ideas and results. |
| **MODELS**<br>(Main Theme: Abstractions and Applications)<br><br>Real-world objects and phenomena can be represented mathematically as models. Models are often approximations or simplifications and have limitations and assumptions. They may be deterministic or probabilistic and could be derived from theory or data. They enable us to describe patterns, analyse situations, predict outcomes and make decisions in those realistic contexts. | **EQUIVALENCE**<br>(Main Theme: Properties and Relationships)<br><br>Equivalence is a relationship that expresses the ‘equality’ of two mathematical objects (e.g. expressions, equations, statements) that may be represented in different forms. The transformation or conversion from one form to another equivalent form is the basis of many manipulations for analysing and comparing them as well as algorithms for finding solutions. |

[1] *Functions*, *Diagrams*, *Models*, and *Equivalence* are also big ideas in the 2020 Secondary Mathematics Curriculum.

| | |
|---|---|
| **TRANSFORMATION**<br>(Main Theme: Operations and Algorithms)<br><br>Transformation refers to changes made to a mathematical object using a clearly defined rule. When an object (e.g. graph, equation, or random variable) is transformed, its properties may or may not change (i.e. invariant). Understanding the nature and effects of these transformations enables us to develop insights into the relationships between the transformed object and the original object and to use these relationships to develop methods to solve problems. | **LIMITS**<br>(Main Theme: Properties and Relationships)<br><br>Limits describe the behaviour of a mathematical object (e.g. a model) that varies with a parameter as the parameter approaches a certain value or infinity. Limits may or may not exist. Both cases provide insights to prove mathematical results, justify the appropriateness of algorithms to obtain approximate value of an exact solution or explain local, long-term or large-scale behaviour. |
| **VECTORS**<br>(Theme: Representations and Communications)<br><br>Vectors are ordered array of numbers. They are higher dimensional generalisation of numbers, which are used to measure or quantify a property of a mathematical object. In its concrete form, vectors are used to describe points, lines and planes in geometry. In its abstract form, a dataset or polynomial can be represented as a vector. Vectors are ways of representing a mathematical object that requires more than one quantity or dimension to specify. | **EXTENSIONS**<br>(Theme: Abstractions and Applications)<br><br>Extensions of a mathematical object, concept or result widen its applicability. Extensions are common in the study of mathematics and is a means by which further abstraction, generalisation and applications can be achieved in mathematics. |

**Mathematics Curriculum Framework**

The central focus of the mathematics curriculum is the development of mathematical problem solving competency. This also includes the curiosity to pose problems and the ability to make conjecture. Supporting this focus are five inter-related components – *concepts*, *skills*, *processes*, *metacognition* and *attitudes*.




- *Mathematical Problem Solving*

Problems may come from everyday contexts or future work situations, in other areas of study, or within mathematics itself. They include straightforward and routine tasks that require selection and application of the appropriate concepts and skills, as well as complex and non-routine tasks that requires deeper insights, logical reasoning and creative thinking. General problem solving strategies e.g. Polya's 4 steps to problem solving and the use of heuristics, are important in helping one tackle non-routine tasks systematically and effectively.

- *Concepts*

The understanding of mathematical concepts, their properties and relationships and the related operations and algorithms, are essential for solving problems. In the A-Level mathematics curriculum, concepts in functions and graphs, sequences and series, vectors, calculus, probability and statistics, and so on, are explored. These content strands are connected and interdependent. At different stages of learning and in different syllabuses, the breadth and depth of the content vary.

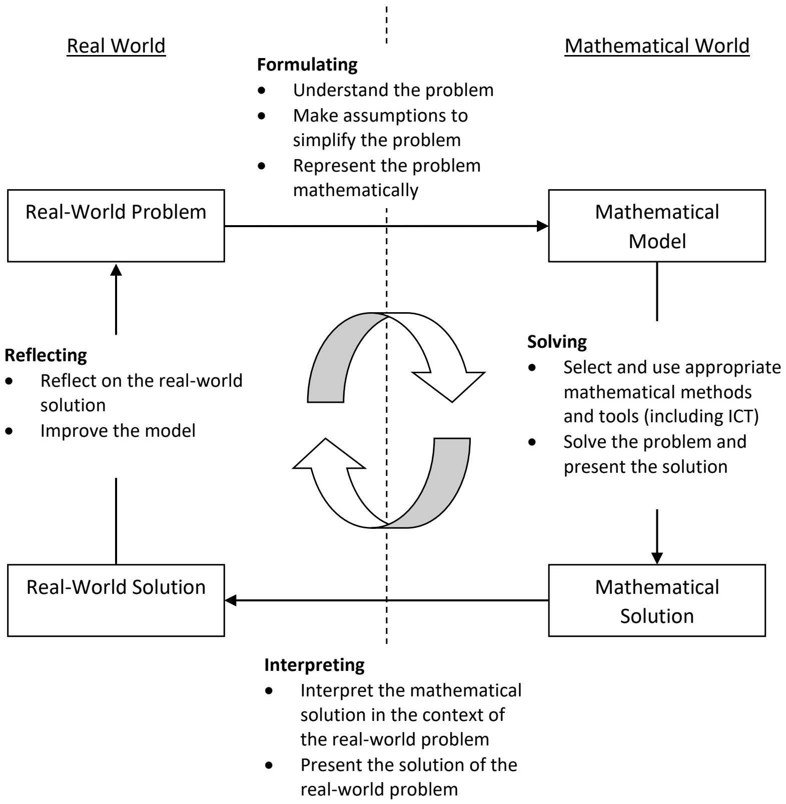
- *Skills*

Being proficient in carrying out the mathematical operations and algorithms and in visualising space, handling data and using mathematical tools (including spreadsheets and graphing applications) are essential for solving problems. In the A-Level mathematics curriculum, operations and algorithms such as *calculation, estimation, manipulation*, and *simplification* are required in most problems.

- *Processes*

Mathematical processes refer to the practices of mathematicians and users of mathematics that are important for one to solve problems and build new knowledge. These include abstracting, reasoning, representing and communicating, applying and modelling. Abstraction is what makes mathematics powerful and applicable. Justifying a result, deriving new results and generalising patterns involve reasoning. Expressing one's ideas, solutions and arguments to different audiences involves representing and communicating, and using the notations (symbols and conventions of writing) that are part of the mathematics language. Applying mathematics to real-world problems often involves modelling, where reasonable assumptions and simplifications are made so that problems can be formulated mathematically, and where mathematical solutions are interpreted and evaluated in the context of the real-world problem. [The mathematical modelling process is shown in the diagram below.]

Mathematical Modelling Process




- *Metacognition*

Metacognition, or thinking about thinking, refers to the awareness of, and the ability to control one's thinking processes, in particular the selection and use of problem-solving strategies. It includes monitoring and regulation of one's own thinking and learning. It also includes awareness of one's affective responses towards a problem. When one is engaged in solving a non-routine or open-ended problem, metacognition is required.

- *Attitudes*

Having positive attitudes towards mathematics contributes to one's disposition and inclination towards using mathematics to solve problems. Attitudes include one's belief and appreciation of the value of mathematics, one's confidence and motivation in using mathematics, and one's interests and perseverance to solve problems using mathematics.

**Mathematics and 21st Century Competencies**

The learning of mathematics creates opportunities for students to develop key competencies that are important in the 21st century, in particular, *Critical, Adaptive and Inventive Thinking*. For example, when students pose questions, justify claims, write and critique mathematical explanations and arguments, they are engaged in not only mathematical reasoning and communication, but also critical thinking. When students devise different strategies to solve an open-ended problem or formulate different mathematical models to represent a real-world problem, they are engaged in inventive thinking. When students vary their approaches to solve different but related problems, they are engaged in adaptive thinking.

As an overarching approach, the A-Level mathematics curriculum supports the development of 21st century competencies (21CC) in the following ways:

1. The content are relevant to the needs of the 21st century. They provide the foundation for learning many of the advanced applications of mathematics that are relevant to today's world.

2. The pedagogies create opportunities for students to think critically, adaptively and inventively, reason logically and communicate effectively, work individually as well as in groups, using ICT tools where appropriate in learning and doing mathematics.

3. The problem contexts raise students' awareness of local and global issues around them. For example, problems set around population, health and sustainability issues can help students understand the challenges faced by Singapore and those around the world.

# SECTION 3:

# H2 FURTHER MATHEMATICS SYLLABUS

Preamble
Syllabus Aims
Content Strands
Applications and Contexts
Content

# 3. H2 Further Mathematics (From 2024)

**Preamble**

Mathematics drives many of the advancements in sciences, engineering, economics and technology. It is at the heart of many of the innovative products and services today. A strong grounding in mathematics is essential for students who aspire to be scientists, engineers or any other professionals who require mathematical tools to solve complex problems.

H2 Further Mathematics (H2 FM) is designed for students who are mathematically-inclined and who intend to specialise in mathematics, sciences or engineering or disciplines with higher demand on mathematical skills. It extends and expands on the range of mathematics and statistics topics in H2 Mathematics and provides these students with a head start in learning a wider range of mathematical methods and tools that are useful for solving more complex problems in mathematics and statistics.

**H2 FM is offered with H2 Mathematics as a double mathematics course.**

**Syllabus Aims**

The aims of H2 FM are to enable students to:

a) acquire a wider range of mathematical concepts and stronger set of mathematical skills for their tertiary studies in mathematics, sciences, engineering and other related disciplines with a heavier demand on mathematics;
b) develop thinking, reasoning, communication and modelling skills through a mathematical approach to problem-solving;
c) connect ideas within mathematics and apply mathematics in the contexts of sciences, engineering and other related disciplines; and
d) experience and appreciate the rigour and abstraction in the discipline.

**Content Strands**

H2 FM comprises 3 content strands, namely, *Algebra and Calculus*, *Discrete Mathematics, Matrices and Numerical Methods*, and *Probability and Statistics*.

a) Algebra and Calculus play a central role in the understanding, development and applications of many branches of mathematics. The strand adds breadth and depth to the topics taught in H2 Mathematics by broadening and deepening the understanding of important mathematical concepts and opening up a wider range of applications that may be useful for the students. It includes mathematical induction, polar curves, and additional topics in complex numbers and calculus. Through these topics, students will be exposed to a wider range of applications in science and engineering, and develop stronger reasoning skills through the writing of mathematical proof.

b) Discrete Mathematics focuses on discrete structures that have many modern real-world applications, especially in computing. Numerical Methods provide useful tools and algorithms to solve problems where exact solutions are not available. This strand adds breadth by introducing problems of discrete nature, in addition to the continuous ones that require calculus, and an 'algorithmic approach' to problem solving in addition to the analytic or algebraic approach that could expose students to basic programming. It includes the study of recurrence relations, matrices and linear spaces and algorithms to solve calculus problems and useful applications such as search engines algorithms. This topic, in particular, offers opportunity for using a computer as a mathematical tool to solve many interesting problems.

c) Probability and Statistics provide the concepts, skills and models to study phenomena where randomness, chance and uncertainty are present. This strand adds breadth and depth to the topics taught in H2 Mathematics by broadening and deepening the understanding of important probability and statistical concepts and offering a larger statistical toolkit that may be useful for future studies and research work. The topics include more statistical and probability models (e.g. general and special continuous random variables such as exponential distribution, additional discrete probability model such as Poisson) and a wider range of applications and statistical methods (e.g. paired sampled tests, non-parametric tests, chi-square tests) that will be useful in areas as far ranging as genetics and politics.

**Applications and Contexts**

As H2 FM is designed for students who intend to specialise in mathematics, science or engineering or disciplines with higher demand on mathematical skills, students should therefore be exposed to the applications of mathematics in science and engineering, so that they can appreciate the value and utility of mathematics in these likely courses of study. The list below illustrates the kinds of contexts that the mathematics learnt in the syllabus may be applied, and is by no means exhaustive.

| **Applications and contexts** | **Some possible topics involved** |
|---|---|
| Kinematics and dynamics (e.g. free fall, projectile motion, orbital motion, collisions) | Functions; Calculus; Vectors |
| Movie graphics | Vectors |
| Optimisation problems (e.g. maximising strength, minimising surface area) | Inequalities; System of linear equations; Calculus |
| Electrical circuits (including alternating current circuit) | Complex numbers; Calculus |
| Population growth (e.g. spread of diseases), radioactive decay, heating and cooling problems, mixing, chemical changes, charging | Differential equations |
| Search engines, cryptography, digital music | Matrices and linear spaces |
| Financial Maths (e.g. banking, insurance) | Sequences and series; Probability; Sampling distributions |

| | |
|---|---|
| Standardised testing | Normal distribution; Probability |
| Market research (e.g. consumer preferences, product claims) | Sampling distributions; Hypothesis testing; Correlation and regression |
| Clinical research (e.g. correlation studies) | Sampling distributions; Hypothesis testing; Correlation and regression |
| Polling | Confidence intervals; Hypothesis testing |
| Genetics | Chi-square tests |

While students will be exposed to applications and contexts beyond mathematics, they are not expected to learn them in depth. Students should be able to use given information to formulate and solve the problems, applying the relevant concepts and skills and interpret the solution in the context of the problem.

**Content**

(Note: Learning Objectives that are italicised are non-examinable.)

| | Topics/ Sub-topics | Content |
|---|---|---|
| **SECTION A: PURE MATHEMATICS** | | |
| **1** | **Algebra and Calculus** | |
| 1.1 | Complex numbers | Include:<br>• complex numbers expressed in the form $r(cos\theta + \mathrm{i}sin\theta)$, $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$<br>• multiplication and division of two complex numbers expressed in polar form and their geometrical interpretations<br>• loci of simple equations and inequalities such as $\|z - c\| \leq r$, $\|z - a\| = \|z - b\|$ and $\arg(z - a) = \alpha$ (excluding loci of $\|z - a\| = k\|z - b\|$, where $k \neq 1$ and $\arg(z - a) - \arg(z - b) = \alpha$)<br>• use of de Moivre's theorem to find the powers and $n$th roots of a complex number, and to derive trigonometric identities |
| 1.2 | Polar coordinates | Include:<br>• simple polar curves (for $0 \leq \theta < 2\pi$ or $-\pi < \theta \leq \pi$ or a subset of either of these intervals; and where $r$ is non-negative throughout the domain)<br>• use of formula $A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 \, \mathrm{d}\theta$ for the area of a sector<br>• arc length of curves defined in polar form |
| 1.3 | Applications of definite integrals | Include:<br>• arc length of curves defined in cartesian form<br>• volume of revolution about the $x$- or $y$-axis for curves defined in cartesian form using discs or shells as appropriate<br>• surface area of revolution about the $x$- or $y$-axis for curves defined in cartesian form |
| 1.4 | Functions of two variables | Include:<br>• functions of two variables and surfaces $z = \mathrm{f}(x, y)$<br>• first order and second order partial derivatives<br>• minimum, maximum and saddle points<br>• normal and tangent planes and local linearisation<br>• directional derivatives and gradient<br>• quadratic approximations<br>• local maximum and minimum problems<br><br>Exclude Jacobian matrix, Lagrange multipliers and constrained optimisation. |

| | Topics/ Sub-topics | Content |
|---|---|---|
| 1.5 | Differential equations | Include:<br>• analytical solution of first order and second order linear differential equations of the form:<br>(i) $\frac{\mathrm{d}y}{\mathrm{d}x}+p(x)y=q(x)$, using an integrating factor<br>(ii) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+a\frac{\mathrm{d}y}{\mathrm{d}x}+by=0$, where $a,\ b\in\mathbb{R}$<br>(iii) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}+a\frac{\mathrm{d}y}{\mathrm{d}x}+by=\mathrm{f}(x)$, where $a,\ b\in\mathbb{R}$ and $\mathrm{f}(x)$ is a polynomial or $pe^{kx}$ or $p\cos(kx)+q\sin(kx)$<br>including those that can be reduced to the above by means of a given substitution<br>• relationship between the solution of a non-homogenous equation and the associated homogenous equation<br>• family of solution curves, *phase lines and slope fields*<br>• exponential growth model<br>• logistic growth model, equilibrium points and their stability, and harvesting |
| **2** | **Discrete Mathematics, Matrices and Numerical Methods** | |
| 2.1 | Recurrence relations | Include:<br>• behaviour of a sequence, such as the limiting behaviour of a sequence<br>• solution of<br>(i) First order linear (homogeneous and non-homogeneous) recurrence relations with constant coefficients of the form $u_n=au_{n-1}+b,\quad a,\ b\in\mathbb{R},\ a\neq 0$<br>(ii) Second order linear homogeneous recurrence relations with constant coefficients<br>including those that can be transformed to the above by means of a given substitution<br>• modelling with recurrence relations of the forms above |

| | Topics/ Sub-topics | Content |
|---|---|---|
| 2.2 | Matrices and linear spaces | Include:<br>• use of matrices to represent a set of linear equations<br>• operations on $3 \times 3$ matrices<br>• determinant of a square matrix and inverse of a non-singular matrix ($2 \times 2$ and $3 \times 3$ matrices only)<br>• use of matrices to solve a set of linear equations (including row reduction and echelon forms, and geometrical interpretation of the solution)<br>• linear transformations and matrices from $\mathbb{R}^n \to \mathbb{R}^m$, where $n, m \leq 3$<br>• eigenvalues and eigenvectors of square matrices ($2 \times 2$ and $3 \times 3$ matrices, restricted to cases where the eigenvalues are real)<br>• diagonalisation of a square matrix **M** by expressing the matrix in the form $\mathbf{QDQ}^{-1}$, where **D** is a diagonal matrix of eigenvalues and **Q** is a matrix whose columns are eigenvectors, and use of this expression such as to find the powers of **M**<br>• linear spaces and subspaces, and the axioms (restricted to spaces of finite dimension over the field of real numbers only)<br>• linear independence and span<br>• basis and dimension (in simple cases), including use of terms such as 'column space', 'row space', 'range space' and 'null space'<br>• rank of a square matrix and relation between rank, dimension of null space and order of the matrix |
| 2.3 | Numerical methods | Include:<br>• location of roots of an equation by simple graphical or numerical methods<br>• approximation of roots of equations using linear interpolation and Newton-Raphson method, *with the aid of a computer*, including cases where each method fails to converge to the required root<br>• iterations involving recurrence relations of the form $x_{n+1} = \mathrm{F}(x_n)$, including cases where the method fails to converge<br>• approximation of integral of a function using the trapezium rule and Simpson's rule, *with the aid of a computer*<br>• approximation of solutions of first order differential equations using Euler method (including the use of the improved Euler formula), *with the aid of a computer* |

| | Topics/ Sub-topics | Content |
|---|---|---|
| **SECTION B: PROBABILITY AND STATISTICS** | | |
| **3** | **Probability and Statistics** | |
| 3.1 | Discrete random variables | Include:<br>• use of Poisson distribution Po($\mu$) and geometric distribution Geo($p$) as probability models, including conditions under which each distribution is a suitable model<br>• mean and variance for Poisson and geometric distributions<br>• additive property of the Poisson distribution |
| 3.2 | Continuous random variables | Include:<br>• probability density function of a continuous random variable and its mean and variance (includes 'piecewise' probability density function)<br>• cumulative distribution function and its relationship with the probability density function<br>• concepts of median and mode of a continuous random variable<br>• use of the result $\mathrm{E}(\mathrm{g}(x)) = \int_{-\infty}^{\infty} \mathrm{g}(x)\,\mathrm{f}(x)\mathrm{d}x$ in simple cases, where $\mathrm{f}(x)$ is the probability density function of $X$ and $\mathrm{g}(x)$ is a function of $X$<br>• uniform distribution and exponential distribution as probability models<br>• relationship between Poisson and exponential distributions |
| 3.3 | Hypothesis testing and Confidence intervals | Include:<br>• formulation of hypotheses and testing for a population mean using a small sample drawn from a normal population of unknown variance using a $t$-test<br>• formulation of hypotheses for the difference of population means, and apply, as appropriate:<br>- a paired sample $t$-test<br>- a test using a normal distribution<br>• contingency tables and $\chi^2$-tests of:<br>- goodness of fit<br>- independence<br>(excluding Yates' correction for continuity)<br>• connection between confidence interval and hypothesis test<br>• confidence interval for the population mean based on:<br>- a random sample from a normal population of known variance<br>- of a small random sample drawn from a normal population of unknown variance<br>- a large random sample from any population |

| | Topics/ Sub-topics | Content |
|---|---|---|
| | | • confidence interval for population proportion (including concept of sample proportion) from a large random sample<br>• interpretation of confidence intervals and the results of a hypothesis test in the context of the problem<br><br>Exclude the use of the term 'Type I error', concept of Type II error and power of a test. |
| 3.4 | Non-parametric tests | Include:<br>• formulation of hypotheses and testing for:<br>- a population median using Sign test<br>- identical probability distributions for two sampled populations in a paired difference design using Wilcoxon matched-pair signed rank test<br>• advantages and disadvantages of non-parametric tests<br><br>Exclude treatment of tied ranks. |

# SECTION 4: PEDAGOGY

Teaching Processes
Phases of Learning
Teaching Towards Big Ideas
Use of Technology
Blended Learning

# 4. PEDAGOGY

**Teaching Processes**

The Pedagogical Practices of The Singapore Teaching Practice (STP) outlines four Teaching Processes that make explicit what teachers reflect on and put into practice before, during and after their interaction with students in all learning contexts.

It is important to view the Pedagogical Practices of the STP in the context of the Singapore Curriculum Philosophy (SCP) and Knowledge Bases (KB), and also to understand how all three components work together to support effective teaching and learning.

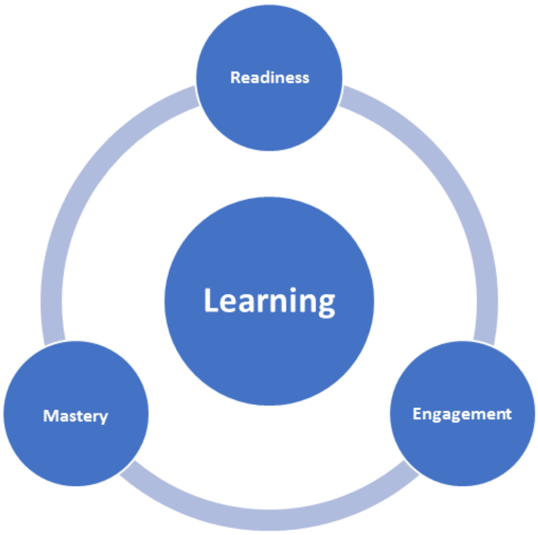
Taking reference from the SCP, every student is valued as an individual, and they have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge, and skills. For learning to be effective, there is a need to adapt and match the teaching pace, approaches and assessment practices so that they are developmentally appropriate.

The 4 Teaching Processes are further expanded into 24 Teaching Areas as shown below.


The Teaching Areas are not necessarily specific to a single Teaching Process. Depending on the context, some of the Teaching Areas could be considered in another Teaching Process. The Teaching Processes are undergirded by a constant cycle of application and reflection.

**Phases of Learning**

The Teaching Areas in STP are evident in the effective planning and delivery of the three phases of learning - *readiness*, *engagement* and *mastery*.




### *Readiness Phase*

Student readiness to learn is vital to learning success. Teachers have to consider the following:

- Learning environment
- Students' profile
- Students' prior and pre-requisite knowledge
- Motivating contexts

### *Engagement Phase*

This is the main phase of learning where students engage with the new materials to be learnt (*Encouraging Learner Engagement*). As students have diverse learning needs and bring with them a wide range of experiences, beliefs, knowledge and skills, it is important to consider the pace of the learning and transitions (*Pacing and Maintaining Momentum*) using a repertoire of pedagogies.

Three pedagogical approaches form the spine that supports most of the mathematics instruction in the classroom. They are not mutually exclusive and could be used in different parts of a lesson or unit. Teachers make deliberate choices on the instructional strategies (Deciding on Instructional Strategies) based on learners' profiles and needs, and the nature of the concepts to be taught. The engagement phase can include one or more of the following:

- Activity-based Learning
- Inquiry-based Learning
- Direct Instruction

Regardless of the approach, it is important for teachers to plan ahead, anticipate students' responses, and adapt the lesson accordingly (Exercising Flexibility).

### *Mastery Phase*

The mastery phase is the final phase of learning where students consolidate and extend their learning. To consolidate, teachers summarise and review key learning points at the end of a lesson and make connections with the subsequent lesson (Concluding the Lesson). The mastery phase can include one or more of the following:

- Motivated Practice
- Reflective Review
- Extended Learning

## Teaching Towards Big Ideas

To enable students to develop a greater awareness of the nature of mathematics, teachers are encouraged to *teach towards big ideas*, where they help students see and make connections among mathematical ideas within a topic, or between topics across levels or strands. An understanding of big ideas can help students develop a deeper and more robust understanding of mathematics and better appreciation of the discipline.

Teaching towards big ideas requires teachers to be conscious of the big ideas in mathematics that are worth highlighting to their students in each syllabus. For each of these big ideas, they must identify the concepts from different topics, levels and strands that exemplify the big idea. Teachers can develop these concepts as they usually do. However, as they teach these concepts, they should find opportune time to make connections between the concepts (horizontal) and the big idea (vertical). This can be done by explaining the connections, or by guiding students to uncover the connections for themselves by asking questions about related small ideas. Students should develop a lens to look at these big ideas in a way that will facilitate learning of related ideas in future.

## Use of Technology

Computational tools are essential in many branches of mathematics. They support the discovery of mathematical results and applications of mathematics. Mathematicians use computers to solve computationally challenging problems, explore new ideas, form conjectures and prove theorems. Many of the applications of mathematics rely on the availability of computing power to perform operations at high speed and on a large scale. Therefore, integrating technology into the learning of mathematics gives students a glimpse of the tools and practices of mathematicians.

Computational tools are also essential for the learning of mathematics. In particular, they support the understanding of concepts (e.g. simulation and digital manipulatives), their properties (e.g. geometrical properties) and relationships (e.g. algebraic form versus graphical form). More generally, they can be used by students to carry out investigation (e.g. dynamic geometry software, graphing tools and spreadsheets), communicate ideas (e.g. presentation tools) and collaborate with one another as part of the knowledge building process (e.g. discussion forum). Getting students who have experience with coding to implement some of the algorithms in mathematics (e.g. finding prime factors, multiplying two matrices) can potentially help these students develop a clearer understanding of the algorithms and the underlying mathematical concepts as well.

**Blended Learning**

Blended Learning transforms our students' educational experience by seamlessly blending different modes of learning. The key intents are to nurture: (i) self-directed and independent learners; and (ii) passionate and intrinsically motivated learners.

Blended Learning provides students with a broad range of learning experiences as shown in the diagram below. An aspect of Blended Learning is the integration of *home-based learning (HBL) as a regular feature of the schooling experience*. HBL can be a valuable complement to in-person schooling. Regular HBL can equip students with stronger abilities, dispositions and habits for independent and lifelong learning, in line with MOE's Learn for Life movement.


**Examples of Blended Learning Experiences**

HBL Days also provide the dedicated time and space for students to actively discover their interests and plan how they should go about pursuing them. Student-initiated learning (SIL) enables students to exercise agency, explore their interests and passions, and learn within and beyond the curriculum.

There are three broad types of SIL activities, namely, school-curated, student-initiated with school facilitation and full student-initiated. Depending on student readiness (e.g. age, disposition, etc.), schools can provide some options for student-initiated learning as scaffolds for those who prefer more guidance at the start, always ensuring that students have agency and choice over what they want to learn. Examples of SIL for A-Level Mathematics are reading a book from the popular maths genre, investigating a problem of interest using open data sources and learning to code.

# SECTION 5: ASSESSMENT

Formative and Summative Assessments
National Examinations

# 5. ASSESSMENT

**Formative and Summative Assessments**

Assessment is an integral part of the teaching and learning. It can be formative or summative or both. It must be fit-for-purpose.

Formative assessment or Assessment for Learning (AfL) is carried out during teaching and learning to gather evidence and information about students' learning. The *purpose* of formative assessment is to help students improve their learning and be self-directed in their learning. In learning of mathematics, just as in other subjects, information about students' understanding of the content must be gathered *before*, *during* and *after* the lesson. This information should inform the starting point of teaching, the development of the concepts, and the remedial actions that may be necessary.

The purpose of summative assessment or Assessment of Learning (AoL), such as tests and examinations, is to measure the extent to which students have achieved the learning objectives of the syllabuses. It often takes place after learning has been completed, for example, after a topic or a series of topics or at the end of a semester or year. Information from summative assessments can also be used formatively, for instance, to help students close learning gaps and decide on steps which they can take to improve their learning.

The outcomes of the mathematics curriculum go beyond just the recall of mathematical concepts and skills. Since mathematical problem solving is the focus of the mathematics curriculum, assessment should also focus on students' understanding and ability to apply what they know to solve problems. In addition, there should be emphasis on processes such as reasoning, communicating, and modelling.

The overarching objectives of assessment should focus on students':

- understanding of mathematical concepts (going beyond simple recall of facts);
- ability to reason, communicate, and make meaningful connections and integrate ideas across topics;
- ability to formulate, represent and solve problems within mathematics and to interpret mathematical solutions in the context of the problems; and
- ability to develop strategies to solve non-routine problems.

Assessment provides feedback for both students and teachers.

- Feedback from teachers to students informs students where they are in their learning and what they need to do to improve their learning. The feedback must be timely and should focus on both strengths and weaknesses of the work done. Additionally, feedback should include ideas on how students can move forward in their learning.
- Feedback from students to teachers comes from their responses to the assessment tasks designed by teachers. They provide information to teachers on what they need

to do to address learning gaps, how to modify the learning activities students engage in, and how they should improve their instruction.
- Feedback between students is important as well because peer-assessment is useful in promoting active learning. It provides an opportunity for students to learn from each other and also allows them to develop an understanding of what counts as quality work by critiquing their peers' work in relation to a particular learning outcome.

**National Examinations**

The first year of examination of H2 Further Mathematics is 2025.

The assessment objectives (AOs), which reflect the emphases of the syllabus and describe what students should know and be able to do with the concepts and skills learned, is shown below.

| Assessment Objectives | Descriptors |
|---|---|
| AO1 | **Use mathematical techniques and procedures**<br>• Recall facts, formulae and notation and use them directly.<br>• Read and use information from tables, graphs, diagrams and texts.<br>• Carry out straightforward mathematical procedures. |
| AO2 | **Formulate and solve problems including those in real-world contexts**<br>• Select relevant mathematical concept or strategy to apply.<br>• Formulate problems into mathematical expressions or models.<br>• Integrate mathematical concepts to solve mathematical problems.<br>• Translate between equivalent forms of mathematical expressions or statements.<br>• Interpret results in the context of a given problem. |
| AO3 | **Reason and communicate mathematically**<br>• Explain the choice of mathematical models or strategies.<br>• Make deductions, inferences and generalisations.<br>• Formulate conjectures and justify mathematical statements.<br>• Construct mathematical arguments and proofs. |

### *Scheme of Examination Papers*

| Syllabus | Scheme of Examination Papers |
|---|---|
| H2 Further Mathematics (9649) | There will be two 3-hour papers, each carrying 50% of the total mark, and each marked out of 100, as follows:<br><br>**PAPER 1 (3 hours)**<br>A paper consisting of 10 to 12 questions of different lengths and marks based on the Pure Mathematics section of the syllabus.<br>There will be one question on the application of Mathematics in real-world contexts, including those from sciences and engineering. The question will |

| | |
|---|---|
| | carry at least 12 marks and may require concepts and skills from more than one topic.<br><br>Candidates will be expected to answer **all** questions.<br><br>**PAPER 2 (3 hours)**<br>A paper consisting of two sections, Sections A and B.<br>**Section A** (Pure Mathematics – 50 marks) will consist of 5 to 6 questions of different lengths and marks based on the Pure Mathematics section (i.e. Algebra and Calculus, and Discrete Mathematics, Matrices and Numerical Methods) of the syllabus.<br>**Section B** (Probability and Statistics – 50 marks) will consist of 5 to 6 questions of different lengths and marks based on the Probability and Statistics section of the syllabus.<br><br>There will be one question in Section B on application of Mathematics in real-world contexts, including those from sciences and engineering. The question will carry at least 12 marks and may require concepts and skills from more than one topic.<br><br>Candidates will be expected to answer **all** questions. |

Further information and details on the national examination are available on the SEAB website.