

## **Knowledge Building with Algebra for a Secondary Two Express Class**

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### **Abstract**

The teaching of Mathematics in Singapore schools typically involves transmissive practices of mere reproduction of knowledge to learners. Teachers tend to follow a procedural paradigm that emphasises on learning computational skills as illustrated in Mathematics textbooks. The concern is that such practices do not provide pupils with sufficient opportunities for developing their conceptual understanding (Tall, 1989; Mok, Johnson, Cheung & Lee, 2000). This action research is focused on fostering a collaborative Knowledge Building community at Coral Secondary for 40 Secondary Two Express pupils to learn algebra using *Knowledge Forum* as a supportive technological tool. A pre- and post-test consisting of open-ended algebraic questions were administered to pupils to collect data on their conceptual understanding. Based on pupils' participation on the Knowledge Forum and their performance on the algebra tests, preliminary results show an improvement in pupils' conceptual understanding of algebra.

### **Introduction**

The teaching of Mathematics in Singapore schools is mainly transmissive and with an emphasis on computational skills. Together with a heavy syllabus content coverage, pupils may not have sufficient time during lesson to grapple with understanding concepts, discuss solutions and to solve problems. That is, current classroom practices do not encourage a revisit of previous learning or allow for extended discussion and differentiated learning, and this may result in pupils having weak retention of content knowledge or inability to connect and transfer learning.

A trend towards pupil-centered learning based on instructional strategies like cooperative learning, project work, problem-based learning, etc is now underway. To thrive in a knowledge economy our youths need to learn how to manipulate knowledge and ideas and also to create new knowledge rather than be passive recipients. We need to nurture the inquisitive nature of our young children and develop them into mature creators of knowledge through Knowledge Building. While didactic and pupil centered approaches still play a role, more needs to be done to develop young minds.

Knowledge Building, a constructive process that calls for deep constructivism, requires an individual to chart his own learning, toy with ideas and improvise

experimentation to test out hypothesis through collaboration within a learning community (Scardamalia, 2000; Scardamalia & Bereiter, 1999; Scardamalia & Bereiter, 2002). Some studies have suggested the potential of technology for developing pupils' thinking. One such report is ImpacT that looked at the impact of information technology (IT) on children's educational achievements in primary and secondary schools in England and Wales (Johnson, Watson, & Cox, 1994). Using technology application such as Knowledge Forum, participants' elaborations over time provide a record of their advances of ideas in a scholarly discipline. The use of Knowledge Forum as an electronic discourse in the learning of Mathematics has not been widely researched and this could be partly due to the notion that Mathematics is absolute truth and leave no room for discussion.

As a group of Mathematics teachers, we are interested in exploring how a Computer-Supported Collaborative Learning (CSCL) environment can be used to support secondary two pupils' conceptual understanding of algebra and to help improve their knowledge application.

## **Methodology**

### *Participants*

Forty Secondary Two Express pupils from class 2E1 participated in this study. 22 of them were girls and 18 were boys. Based on their Mathematics achievement in Secondary One, the class is considered mixed ability. Ninety percent of the pupils had Internet access at home and those without home access could work in the school's computer labs or at their friends' homes. All of them had knowledge of the Internet and also some experience with online synchronous chats.

### *Knowledge Forum*

The Knowledge Forum application provided the CSCL environment for pupils to engage in a discourse on algebra. It is a technological tool specifically developed to support Knowledge Building. Knowledge Forum is an asynchronous discussion board providing a multimedia community space for participants to contribute theories, working models, plans, evidence, reference material, etc - a structure similar to mind-mapping to support knowledge building (Scardamalia, 2004). The analysis tool kit in Knowledge Forum can be used to analyse the discussion postings. An accompanying scoring rubric developed by the team is used to assess pupils' participation based on

their number of postings and the quality their discussion. The band descriptors range from inactive to very active (see Appendix A).

*Pre- and Post-Test*

An algebra test consisting of a total of four open-ended questions was constructed and it was used both as a pre- and post-test for assessing pupils' conceptual understanding. The four questions are:

1. Using your own words, explain the term “algebra” based on your own understanding.
2. Explain the following concepts related to basic algebra. Provide examples and mathematical expressions when necessary.
  - a. Explain the differences between algebraic equation and algebraic expression.
  - b. Explain the meaning of polynomials, coefficients, variables and constants as used in algebra.
3. Explain clearly the basic steps involved in using algebra to solve a word problem. Use the word problem provided below to articulate your steps.

Problem:

A man travels from A to B at 4 km/h and travels back from B to A at 6 km/h. If he takes 45 minutes to complete the whole journey, what is the total distance travelled?

(Answer: 3.6 km)

4. What do you understand by the terms “factorization” and “expansion” of algebraic expression?

A rubric adapted from Entwistle (2000) work on levels of understanding was used for scoring pupils' responses based on their level of conceptual understanding of each question (see Appendix B). The band descriptors range from ‘mentioning’ as the lowest level to ‘conceiving’ as the highest level of understanding. These scores from four questions were computed and used for analysis.

## *Procedures*

### *Pilot Study*

A pilot test of the Knowledge Forum was conducted between January and early March 2004. The purpose was to allow a Knowledge Building culture to evolve and to familiarise pupils with the use of Knowledge Forum. Pupils were introduced to the idea of Knowledge Building through several knowledge building principles (real ideas, authentic problems, improvable ideas, idea diversity, epistemic agency, community knowledge, collective responsibility and democratizing knowledge) and taught the basic functions in Knowledge Forum over six sessions in class and four sessions in the computer laboratory. This familiarisation phase was conducted concurrently with classroom teaching and the Knowledge Forum was used as a supporting tool to complement their learning.

Several interesting observations were made about pupils' behaviour. They used the Knowledge Forum more like a real-time chat with only single word or one liner type of postings and unsure of how to use scaffolds to guide discussions. They had arguments with one another and did not respect each other views. Also, they faced problems using Maths Type 5 software to create mathematical symbols. Most of the problems were resolved or minimised before the main study.

### *Main Study*

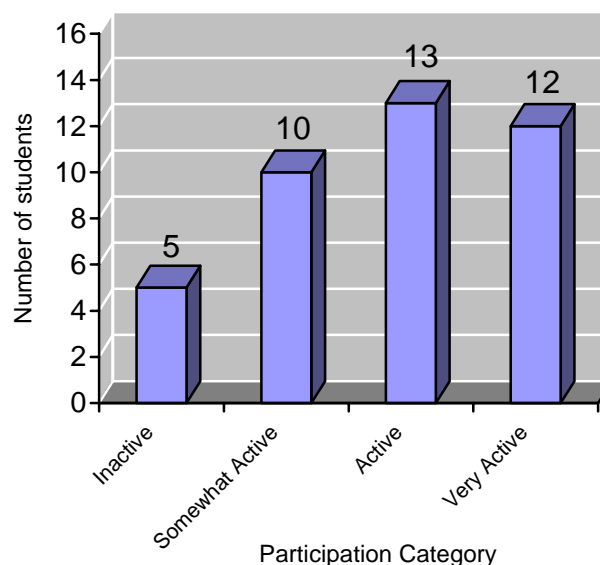
The main study was conducted between Mid-March and May 2004. A pre-test was first administered to the pupils to assess their conceptual understanding. The scheme of work for these chapters was shared with the pupils to assess their understanding of algebra concepts. Pupils had face-to-face discussions in class as well as in the school's computer labs and online discussions on Knowledge Forum after school. They had the freedom to work in small groups or individually on areas of their interest within a chapter in class. Some interesting issues from Knowledge Forum were brought into class for further discussions and continued further back in Knowledge Forum. A member of the team who was subject teacher of the class acted as facilitator and e-moderator throughout the knowledge building process, guiding pupils but not intruding and directing discussions as much as possible. Pupils designed their own tasks like discussing exercises from the textbook, experimentation with concepts, rules, methods of solving, etc. In the school's computer labs, pupils could explore the Internet to read up on relevant topics or discussed with their peers on postings in

Knowledge Forum. They monitored their own progress and understanding through maintaining an individual reflection logs on a regular basis.

A post-test of the open-ended questions was administered to the pupils after the intervention. Pupils' responses from the pre- and post-test were scored and discussion obtained from the Knowledge Forum were analysed.

## Results

Figure 1 shows pupils' participation in the Knowledge Forum. Five pupils were inactive, ten pupils somewhat active, thirteen pupils active and twelve pupils very active in the discussion on Knowledge Forum.



**Figure 1. Participation in Knowledge Forum**

Sample postings by pupils from inactive and very active groups were analysed to ascertain the differences in conceptual understanding between the two extreme groups. The detailed notes by these pupils are shown in Appendix C.

For pupils that were inactive, one general observation is that they have less than 10 postings. Their postings do not seem to indicate engagement in any discussions and are sporadic in nature. Some examples are “*How do we know which type of perfect square we should use?*”, “*I couldn't understand what you mean.. It is really confusing!*”, “*why must we cross multiply....why can't we just multiply it straight? i am confused..*”, etc. They are mainly very confused and are just waiting for others to explain to them without making an effort to think deeper themselves to construct knowledge.

For pupils that were very active, it was observed that they have numerous postings. This group of pupils were also observed to have moved from a single liner type of postings to a more coherent piece of work as they progress through Knowledge Building. Their postings clearly showed that they constantly developed deeper conceptual understanding through a series of self-reflections, synthesis of ideas, hypothesis, conjectures and knowledge creation by collaborating with their peers. Some examples are *“I think they are not really connected. Firstly, expansion and factorisation deal with algebra....But look at the methods...they are different. But the grouping and extracting common factors may be used in indices.... My theory may be proven wrong or partially right. So please give your comments”*, *“This is the result but I didn't satisfy you enough cause.... There are lots of possibilities to get (x) and (y) such as.... Hence, from here I will let you carry on. I gave you the ideas I have, so try to find the ratio .If any way I find the exact answer, I will contribute.”*, *“but in this case, if we use the cross -multiply method, it does not work... it does not give us the correct answer, and yin liang have check out that the answer is 1/2, so I think other methods like grouping? i am not sure too.. please correct me if I am wrong”*, etc.

Pupils' level of understanding on all the four open-ended questions on algebra was analysed. Figure 2 and Figure 3 show the results for the pre- and post-test respectively. The pre-test results show that most pupils were at the 'describing' level for question 1, 'describing' and 'relating' levels for question 2, 'describing' level for question 3 and, 'mentioning' and 'describing' levels for question 4.

Compared to the pre-test, pupils improved in their level of understanding in all questions. More pupils reached the 'describing' and 'relating' levels for question 1, the higher levels of 'relating', 'explaining' and 'conceiving' for question 2, the 'relating' level for question 3 and the 'describing' and 'explaining' levels for question 4.

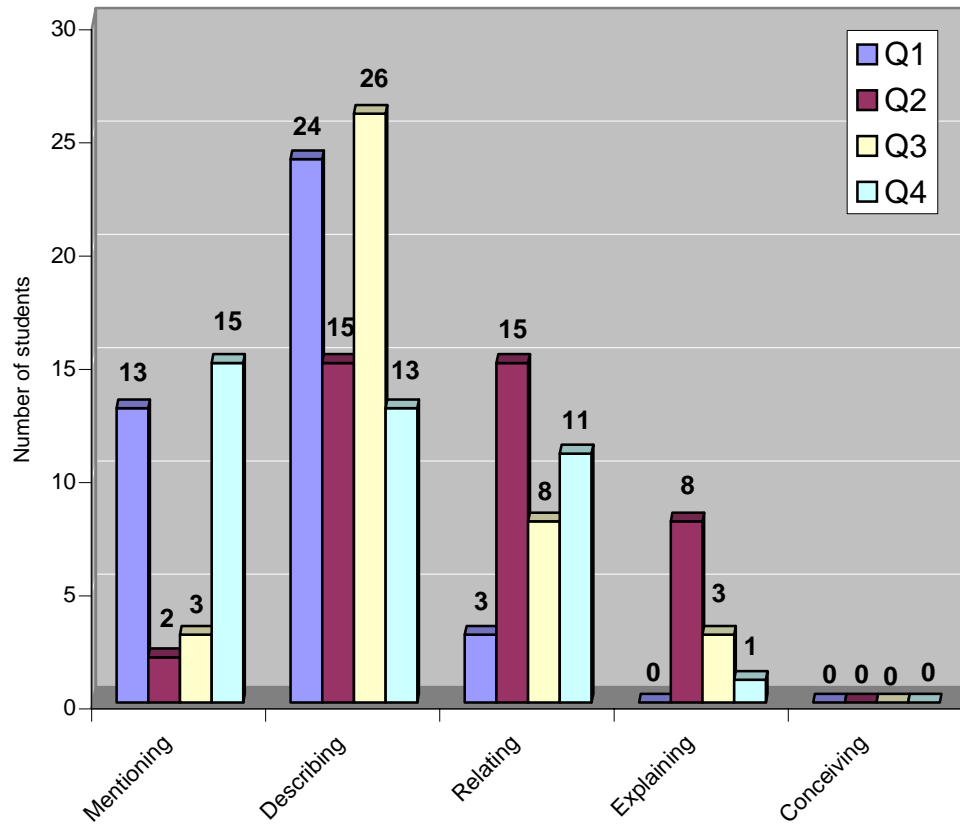


Figure 2. Pre-Test Results by Level of Understanding

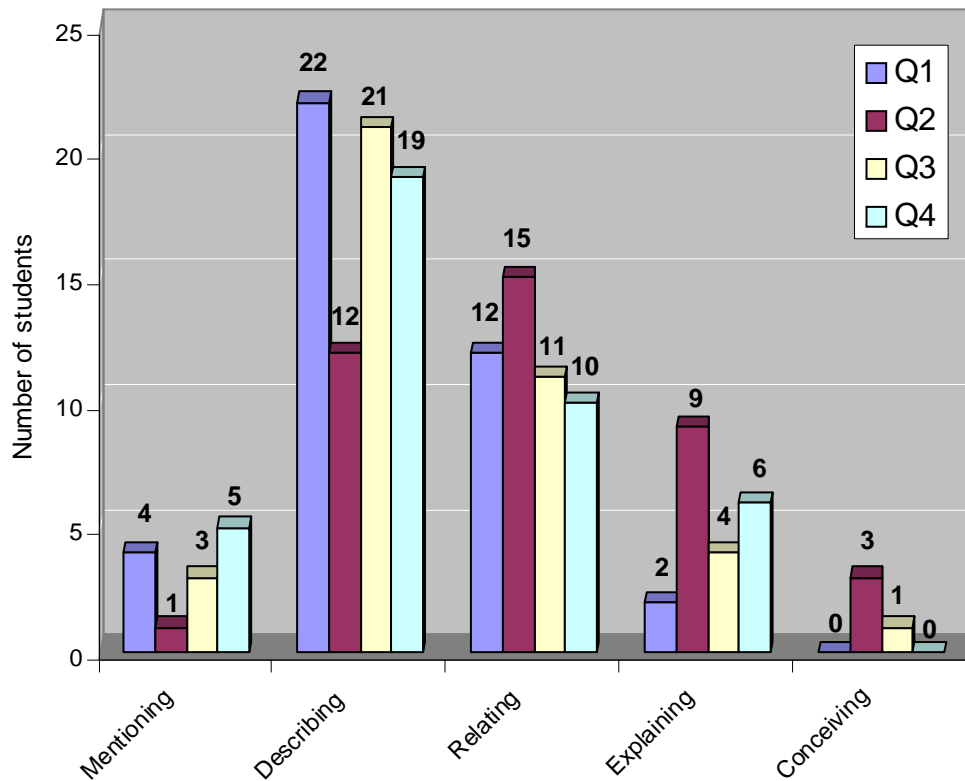


Figure 3. Post-Test Results by Level of Understanding

## **Discussions and Conclusion**

This study has focussed on the effect of Computer-Supported Collaborative Learning (CSCL) on pupils' conceptual understanding of algebra. Preliminary results show a trend of better conceptual understanding of pupils after Knowledge Building. That is, active participation in discussions through Knowledge Forum can help improve pupils' conceptual understanding. On the other hand, pupils who are inactive or somewhat active participants tend to remain at mentioning level in the post-test. For some inactive pupils who improved in their conceptual understanding it may be due to their active participation in face-to-face discussions with peers.

Some pertinent problems regarding Knowledge Building and use of a CSCL environment are noted. These include: (a) some pupils were inactive (12.5%) and it was rather difficult to encourage them to participate to reap the benefits of Knowledge Building; and (b) many pupils were unable to synthesize ideas together from the discussion. Solutions to minimise these problems can include having pupils take turns to synthesize ideas and to start a new posting for discussion in a more structured manner – this may help promote greater ownership and interactions.

Introducing Knowledge Building for the learning of algebra has several implications. Firstly, Knowledge Building is an inquiry-based learning approach that promotes intrinsic motivation in pupils to continuously pursue knowledge at their own pace. Secondly, Knowledge Forum provides a non-threatening environment for pupils who are too shy to ask questions in class. Thirdly, collaborative knowledge building within a learning community may improve conceptual understanding and thus make knowledge retention more enduring. Finally, using “Understanding by Design” framework to guide the inquiry process can help re-structure a didactic style of teaching and help support and promote understanding (Wiggins & Mctighe, 2000). While this study demonstrates that Knowledge Building with the support of Knowledge Forum can help promote pupils' conceptual understanding in algebra, further studies will be needed to fine-tune the process of Knowledge Building and to ensure an engagement of pupils in discussion.

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## Appendix A

**Table 1. Rubric for Participation Categories in Knowledge Forum**

<i>Participation Categories in Knowledge Forum</i>		
<i>Category</i>	<i>Categories of Participation</i>	<i>Descriptions</i>
1	<i>Inactive</i>	Less than 10 postings and cursory response without any obvious interactions
2	<i>Somewhat Active</i>	Slightly more than 10 postings with some evidence of interactions
3	<i>Active</i>	Adequate postings with evidence of interactions, questionings and building knowledge with peers
4	<i>Very Active</i>	Sufficient postings with strong evidence of interactions, questionings and knowledge building with structured, independent argument

## Appendix B

**Table 2. Scoring Rubrics for Levels of Understanding**

<i>Levels of Understanding</i>		
<i>Level</i>	<i>Categories of Understanding</i>	<i>Descriptions</i>
1	<i>Mentioning</i>	Incoherent bits of information without any obvious structure
2	<i>Describing</i>	Brief descriptions of topics derived mainly from material provided
3	<i>Relating</i>	Outline, personal explanations lacking detail or supporting argument
4	<i>Explaining</i>	Using relevant evidence to develop structured, independent argument
5	<i>Conceiving</i>	Individual conceptions of topics develop through continuing reflection

## Appendix C

Sample notes extracted from Knowledge Forum

Postings from pupils in participation level 1:

- Pupil A: *"Dividend= divisor X quotient + remainder"*  
*"please look carefully in the second statements.... i sure there's a few mistakes.."*
- Pupil B: *"Expansion can be solved using: (a) distributive law...."*  
*"How do we know which type of perfect square we should use?....."*
- Pupil C: *"i will try to find out for you... i think must factorise..."*  
*"I think there should be some more methods to solve...Maybe we can use...."*  
*"So sorry... I couldn't understand what you mean.. It is really confusing!"*  
*"I dun understand with your method. Why..... (5-y)? Can you explain??"*  
*"I think that Indices, factorisation and expansion has a little linking....."*
- Pupil D: *"the equation  $y=ax^2$  .. the a cannot be equal to 0 .. why issit so ???"*  
*"i know it is a method but can i multiply it straight or must i .....???"*  
*"why must we cross mutiply....why cant we just multiply it straight? i confused.."*  
*"wads linear equations? i still cannot understand clearly .."*
- Pupil E: *"my ans. is the same as chen hong's as he was the one who help me solved this qns."*  
*"... i dun hav any examples....help me think of some examples....."*  
*"How do you express problems in the form of simultaneous linear equations?"*  
*"Musaddiq, you are on the road to understand this so try harder."*  
*"Can anyone help me simplify the equation?"*  
*"not entirely only that method. we can also use other methods"*  
*"How do you do equations that involves many sums?"*

Postings from pupils in participation level 4:

- Pupil A:  
*"As we all now, we can just cancel out the common factors in the fraction provided if the number before it is a multiplication"*  
Eg.  $\frac{2(a+b)}{5(a+b)} = \frac{2}{5}$   
*Ok...whether we can cancel out the common factors if there is a division?*  
Eg.  $\frac{2(\text{divide})(a+b)}{5(\text{divide})(a+b)} = ???...$   
*"I think they are not really connected. Firstly, expansion and factorisation deal with algebra....But look at the methods.....,they are different. But the grouping and extracting common factors may be used in indices.....Hence, I can say that they are.... My theory may be proven wrong or partially right. So please give comments.."*  
*"This is what I obtained. I have a question.....Is it just simply 3+3 or other rules.....This uses distributive law....Just tell me the reasons..... I know thanks....but"*

still a little unsure. Is it because of pascal triangle or something. But I still don't think because of pascal. Look at the problem again. The answer as showned.  $(p-3q+r)(p+3q-r)$ ....., so how to get  $(6a)$ . Anyway how did....”

“It seems impossible. I tried to get the common factors of 6 which are....not sure if can find by fractions or decimals. For now ,I used the methods.....Which we all recently learnt....I also noticed that in the question..... But I loved to know if there are possibilities..... I am keen to learn and learn from mistakes.”

“This is the result but I didn't satisfy you enough cause.... There are lots of possibilities to get  $(x)$  and  $(y)$ .such as..... $y$  could be..... Hence, from here I will let you carry on. I gave you the ideas I have ,so try to find the ratio .If any way I find the exact answer ,I will contribute OK!!!”

“<Putting our knowledge together> is that only quadratic equations require the RHS =zero. In Chapter 8,the methods use to solve simultaneous equation such as elimination and substitution method do not need to do so. Well after I further understood Chapter 8 it seems like it is not necessary do make the RHS to zero.”

Pupil B:

“shuLing,  $m$  to the power of 0 is 1, remember ?..this is one of the indices law.... as we know that one of de indices of law is.....so did i clear all your doubts?”

“but in this case, if we use the cross -multiply method, it does not work.. it does not give us the correct answer, and yin liang have check out that the answer is  $1/2$ , so i think other methods like grouping? i am not sure too.. please correct me if i am wrong”

”...consecutive positive number..they did not mention is even or odd number... so we only need to plus 1.. if u take the from the maths textbk pg84.. example4.. that question is.... that is why..... u understand? if u still do not understand u can find me and we can have face to face discussion”

“according to what mr teo had said, i think i am wrong...”

“<Putting our knowledge together> is that the simultaneous linear equation i post up is easier to solve it by using elimination method as we just use.... the 2nd equation..... i do not think we need to do both methods.... graphical method is draw on the graph paper right? anyway, i am not sure how does it works”

“i have some steps to guide you to solve questions involving using quadratic equations

step1: Let  $x$  be the unknown number to be found.

sStep 2: use the information given to form a quadratic equation.

.....

step 5: check that the values are acceptable .”

“summary for solution for ex6c question 3.. basically.. the method use is trial and error.. for quadratic equations.. there will be two answer.. normally we will take the positive answer... and if all the terms have..... divide everything by that common factor so that it is easier....”

“summary of ex6c question8.. in this question.. what is the most important thing is make RHS to be zero den factorise LHS

example :  $x^2 + 3x - 5x - 15 = 20$

$$x^2 + 2x - 35 = 0$$